A UNIVERSAL METRIC FOR PRICING CREDIT EXPOSURES

by

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This version: January 12, 2007

Abstract

As some CDOs are quoted on the markets, it is possible to imply the pricing function according to the seniority of the exposure. In this article we show how to extract this information to have an efficient way to price a given bank's portfolio that is valuated under a (historical) internal portfolio model. We also show how to compute risk contributions and hurdle rates for each operation on a bank portfolio.

Acknowledgment: See later

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1 INTRODUCTION

Corporate finance models often suffer from a lack of data to make calibration. In the field of banking corporations, we are lucky to deal with financial assets on the assets side and with issuance of equity and debt on the liabilities side. To make a long story short, a bank's portfolio is funded by issuing huge Collateralized Debt Obligations (CDOs). The quotations of standardized CDOs on the I-Traxx CDS indices for instance have opened new doors for calibrating bespoke and tailor-made CDO tranches. In this paper, we describe the general methodology to accomplish this calibration and we apply it to the bank's portfolio. In particular, we propose a new approach to solve the loan pricing problem and the computation of the business lines cost of risk.

As the bank's structure of capital is similar to the structure of a CDO, we are able to bridge corporate finance and market observations. In the case of market CDOs, the market prices reflect the investors' risk aversion and pricing function as a function of exposure seniority (or confidence level). We show that the market implied risk aversion is closely related to the cost of risk of CDO tranches. The main hypothesis in this paper is to assume that the market CDO investors have the same risk aversion functional as the investors in the bank's structure of capital. Since the underlying portfolios of assets for I-Traxx and the bank are completely different, we argue that the investors in both portfolios are going to price the tranche of risk corresponding to a given loss probability the same way. At the end of the day, both portfolios are market to the market, and there exist a metric in which they are identical.

The paper is structured as follows. In section 1, we show that the cost of risk is a coherent risk measure, and in particular, is written as a weighted sum of value at risks. We express the relationship between this weight function and the "risk neutral" probability measure. An illustration with homogeneous portfolios is shown in section 2. Section 3 is devoted to the calibration of the weight function upon CDO market data. In section 4 and 5, we present some applications to the credit risk management of a bank's portfolio.

2 COST OF RISK AS A COHERENT RISK MEASURE

In this section, we show that the price of risk on a credit portfolio is necessarily related to coherent risk measures. Let us consider a credit portfolio whose loss is a random variable called L. We call F the distribution function of the loss random variable on the underlying portfolio, and f its loss density. We will show in the next section how to infer the "risk neutral" or market implied distribution function of the variable L when its underlying pool is quoted on the market under a CDO structure. The mark to market price of the portfolio includes a risk premium, which means that the price of risk is larger than the average risk. As the price of risk is closely related to a mark to market valuation, we argue that there is a "risk neutral" (distorted) probability measure $\mathbb{Q}$ relating the losses on the portfolio and the price of risk in the following way:

$$Price\ of\ Risk = \mathbb{E}_{\mathbb{Q}}\left[L\right] = \mathbb{E}\left[L\varphi(L)\right] \tag{1}$$

$\mathbb{E}$ denotes the expectation under the usual historical probability. This probability is distorted through the function φ, where φ needs to be calibrated on both market and historical data. φ is the Radon-Nicodym derivative $\frac{d\mathbb{Q}}{d\mathbb{P}}$ and in particular, $\mathbb{E}\left[\varphi(L)\right] = 1$.

[NOTE:]

It may be more complex than that: we should have only the following equality

$$\mathbb{E}\left[\frac{d\mathbb{Q}}{d\mathbb{P}}\middle|\mathcal{F}_L\right] = \varphi(L). \tag{2}$$

[END NOTE]

The price of risk is itself a risk measure. Artzner et al. ([2]) have exhibited coherence conditions of a risk measure in order to avoid undesirable drawbacks. In his paper ([6]), Wang shows that the expected loss under a distorted probability measure leads to a coherent risk measure when some conditions on the Radon-Nikodym derivative are satisfied. This is interesting because it bridges the risk management world to a "risk neutral" world, and it guarantees good properties of the price of risk measure.

The fact that we can express the price of risk of the whole portfolio in terms of an expected loss under a distorted probability measure, leads us to consider the price of risk of the whole portfolio as the sum of the prices of risk of each Infinitely Thin Tranches (ITT). This intuition comes from the varying probability weights according to the loss level. This trick is now of a common use in the field of securitization because knowledge of each ITT means knowledge of any capital structure of the portfolio (see [4]). The loss of the ITT located at level l is simply the binomial random variable $\mathbb{1}_{\{L>l\}}$, making the calculation on ITT often very easy, and calculations on any structured exposure is the result of integrals onto IITs. The expected loss of the ITT is equal to the survival function $\mathbb{P}[L > l] = 1 - F(l)$, and its price of risk $POR(l) = \mathbb{Q}[L > l]$is equal to the "risk neutral" survival function. After summation, we recover the total price of risk:

$$POR = \int_0^{L_{Max}} POR(l)dl \tag{3}$$

It is then equivalent to know the ITT price of risk function and the distorted probability measure $\mathbb{Q}$. Deriving with respect to l the equality:

$$POR(l) = \mathbb{Q}[L > l] = \int_l^{L_{Max}} f(l)\varphi(l)dl, \tag{4}$$

we get the following relationship between the probability distortion function and cost of risk for each ITT:

$$\varphi(l) = -\frac{1}{f(l)} \frac{\partial POR(l)}{\partial l} \tag{5}$$

When the loss on the portfolio has a risk-neutral distribution function $f_{\mathbb{Q}}$ (that is the opposite of the derivative of the local Price Of Risk $POR(l)$), the previous equality writes more simply under the following form:

$$f_{\mathbb{Q}}(l) = \varphi(l) f_{\mathbb{P}}(l) \tag{6}$$

We can also rewrite the price of risk of the portfolio in the quantiles space, and this leads to a deep understanding of the risk measure associated to the portfolio. With the change of variables $\alpha = F^{-1}(l)$ or equivalently $l = VaR_\alpha(L)$, we get:

$$POR = \int_0^1 \tilde{\varphi}(\alpha) VaR_\alpha(L) d\alpha, \tag{7}$$

where $\tilde{\varphi}(\alpha) = \varphi \circ F^{-1}(\alpha)$.The function $\tilde{\varphi}$ is the weight the investor assigns to a given confidence level in his pricing. It is directly related to his risk aversion at the threshold α . Following Wang, we can show that the function is the derivative of a distortion function. The coherence axioms are satisfied if the distortion function is continuous. In particular, in our case, the price of risk is a coherent measure if the primitive of the weight function $\tilde{\varphi}$ is continuous. [**Talk more about that particular point. Insist on it later once $\tilde{\varphi}$ has been computed.**]This shows that mark to market pricing naturally leads to coherent measures of the price of risk.

[**NOTE :**]

1. How to define φ when $f_{\mathbb{P}}(l) = 0$ and $f_{\mathbb{Q}}(l) \neq 0$ (senior tranches).
2. $\tilde{\varphi}(\alpha) \neq \frac{VaR_\alpha^{\mathbb{Q}}(L)}{VaR_\alpha^{\mathbb{P}}(L)}$

Shouldn't we define $\tilde{\varphi}$ as $\tilde{\varphi}(\alpha) = \frac{VaR_\alpha^{\mathbb{Q}}(L)}{VaR_\alpha^{\mathbb{P}}(L)}$. This would be more easy and understandable. It would also solve point 1.
Note also that if point 1 exists, it shows (proves) that $\mathbb{P}$ and $\mathbb{Q}$ cannot be equivalent measures.
[**END NOTE**]

When the weight function is below 1, the market underestimates the occurence of the α-level scenario, while this occurence is overweighted when $\tilde{\varphi}$ is above 1. High values for $\tilde{\varphi}$ for high quantiles are in lines with market prices far above historical losses on senior tranches.

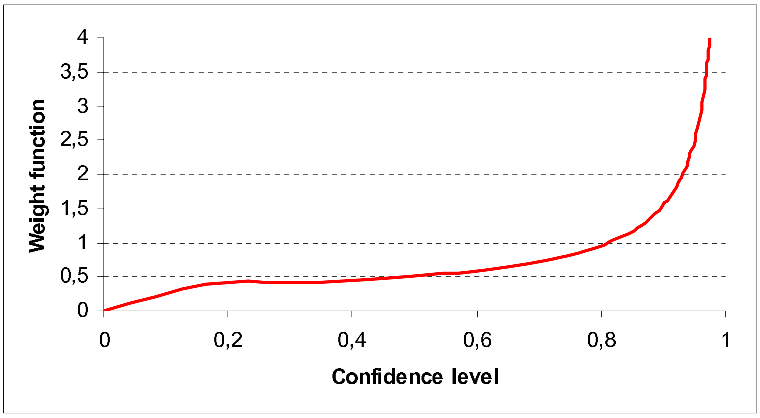

Figure 1: Weight function for historical and risk neutral Vasicek densities.

The differences between historical and risk-neutral worlds are summarized in the table below.

Summary: historical vs. cost of risk.

	Historical	Risk-Neutral
Prob. Dens. Function	$f_{\mathbb{P}}$	$f_{\mathbb{Q}} = \varphi\ f_{\mathbb{P}}$
Cost of ITT(l)	$\mathbb{P}[L > l]$	$\mathbb{Q}[L > l]$
Total Cost of Risk	$\mathbb{E}_{\mathbb{P}}[L]$	$\mathbb{E}_{\mathbb{Q}}[L] = \mathbb{E}_{\mathbb{P}}[\varphi(L)L]$
Value-at-Risk	$VaR_{\alpha}^{\mathbb{P}}(L);\ \int_{VaR_{\alpha}^{\mathbb{P}}(L)}^{L_{Max}} f_{\mathbb{P}}(l)dl = 1-\alpha$	$VaR_{\alpha}^{\mathbb{Q}}(L);\ \int_{VaR_{\alpha}^{\mathbb{Q}}(L)}^{L_{Max}} f_{\mathbb{P}}(l)\varphi(l)dl = 1-\alpha$

We now explain in the following section how to draw the previous graph in the ideal case of an homogenous and granular portfolio.

3 ILLUSTRATION ON GRANULAR PORTFOLIOS

For infinitely granular homogeneous credit portfolios, Vasicek ([5]) has explicited the loss distribution that depends on three parameters only : the average default rates (p) and losses given default (LGD) on the portfolio's counterparts and the asset correlation ρ between counterparts. The density function writes[3]:

$$V(l,p,\rho) = \sqrt{\frac{1-\rho}{\rho}} \exp\left[-\frac{(1-2\rho)\left(\mathcal{N}^{-1}(l)\right)^2 - 2\sqrt{1-\rho}\mathcal{N}^{-1}(l)\mathcal{N}^{-1}(p) + \left(\mathcal{N}^{-1}(p)\right)^2}{2\rho}\right] \tag{8}$$

If the risk neutral loss distribution is also a Vasicek law with risk neutral parameters $\tilde{p}$ and $\tilde{\rho}$, then the function φ is the ratio between the risk neutral and the historical Vasicek densities:

$$\varphi(L) = \frac{V(l,p,\rho)}{V(l,\tilde{p},\tilde{\rho})} \tag{9}$$

[3]To simplify, we suppose that the Loss Given Default is equal to 1.

In realistic cases, the risk neutral parameters are larger than the historical parameters, and the function $\tilde{\varphi}$ is an increasing function of the confidence threshold α. Figure 1 shows the function for the following parameters: $p = 1\%$, $\tilde{p} = 5\%$, $\rho = 20\%$ and $\tilde{\rho} = 35\%$.

We see that the curve has a plateau around the median. The presence of this plateau depends on the parameters, but we will see that there is also a plateau for real market implied $\tilde{\varphi}$ functions. The plateau means that in some range of confidence thresholds, the marginal risk aversion is equal to zero; this generally happens in the bulk of the loss distribution. Qualitatively speaking, the curves are very similar in a model where the risk neutral correlation is no longer unique but is given by an increasing base correlation as it is the case on CDO markets.

In practice, CDO tranches are priced in terms of local correlation and it is possible to link $\tilde{\varphi}$ with the implied correlation of the CDO. The risk-neutral correlation (or base correlation) represents the risk neutral perception of risk by the market. The base correlation at the loss level l is the unique correlation $\rho_B(l)$ for which the market price of the tranche is given by the Vasicek model with parameters $\tilde{p}$, $\widetilde{LGD}$ and $\rho_B(l)$. As quantile at the α level of the losses on the portfolio under historical probability is:

$$VaR_\alpha(L) = LGD\,\mathcal{N}\left(\frac{\mathcal{N}^{-1}(p) - \sqrt{\rho}\mathcal{N}^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right), \tag{10}$$

and similarly, the VaR under the $\mathbb{Q}$ probability is

$$\widetilde{VaR}_\alpha(L) = \widetilde{LGD}\,\mathcal{N}\left(\frac{\mathcal{N}^{-1}(p) - \sqrt{\rho_B(l)}\mathcal{N}^{-1}(1-\alpha)}{\sqrt{1-\rho_B(l)}}\right), \tag{11}$$

we obtain the value of $\tilde{\varphi}$ for each probability level α:

$$\tilde{\varphi}(\alpha) = \frac{\widetilde{LGD}}{LGD}\,\frac{\mathcal{N}\left(\frac{\mathcal{N}^{-1}(p) - \sqrt{\rho_B(l)}\mathcal{N}^{-1}(1-\alpha)}{\sqrt{1-\rho_B(l)}}\right)}{\mathcal{N}\left(\frac{\mathcal{N}^{-1}(p) - \sqrt{\rho}\mathcal{N}^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right)} \tag{12}$$

FALSE !!!

$\tilde{\varphi}(\alpha) \neq \frac{VaR_\alpha^{\mathbb{Q}}(L)}{VaR_\alpha^{\mathbb{P}}(L)}$

END FALSE

As shown by previous equation, a probability distortion function is applied by the market on the loss distribution function. Instead of working on probability levels, the market distorts correlations which is, as we have exhibited it just above, equivalent. **[Etayer]**

We detail in the following section how to derive from market data the base correlation function, the cost of risk on an ITT, and finally, how to compute the $\tilde{\varphi}$ function.

4 CALIBRATION

Since a few years now, some CDS indices such as I-Traxx and CDX, are quoted, and more interestingly, CDOs on these indices exist. These CDOs are benchmarks for structuring and pricing complex structured credit derivatives. It is thus reasonable to view them as the way investors price structured credit exposures. Our idea in this section is to derive the ITT price of risk on the I-Traxx CDS index. The first obstacle to deal with is that the quoted CDO tranches are far from being ITTs. The four first tranches have size 3% of the pool nominal, but we argue that an interpolation of the base correlation function as a function of the equity size leads to a reasonable first step in calibrating the market implied loss densities and then the risk measure .

Base correlations are often used instead of implied (or compound) correlations because of their stability. Our methodology for computing the ITT price of risk function is a four step procedure:

- Interpolation of the base correlations to get a continuous and differentiable function $\rho_B(l)$.
- Compute the zero-coupons collateralized by the equity tranches $[0, l]$. The price of this zero-coupon bond writes : $l \exp -s(l)T$, where $s(l)$ is the spread corresponding to the $[0, l]$ equity tranche and T is the maturity of the bond and of the underlying portfolio, equal to 5 years. These zero-coupon spreads are outputs of a pricing model. For this study we have implemented the model described in Andreasen et al. (**Write reference here**).
- Compute the zero-coupon bond prices of the ITTs at level l . Since the equity tranche $[0, l]$ is the sum of all the ITTs up to level l, we have the following relationship on zero-coupons at the l level:

$$l \, e^{-s(l)T} = \int_0^l e^{-s_C(x)T} dx, \tag{13}$$

  where $s_C(l)$ is the spread of the zero-coupon associated with the ITT of level l.
- The cost of risk of the ITT at level is the difference between a non-risky zero-coupon and the price of the risky zero-coupon:

$$POR(l) = 1 - e^{-s_C(l)T}. \tag{14}$$

These four steps then lead to the functions φ and $\tilde{\varphi}$ by derivation of the ITTs price of risk function (see equation (5V)).

interpolating the base correlation for low strikes (below the equity tranche detachment point of 3%) is still an open question. However,as these strikes correspond to small values of α, for which the risk aversion of the market is very low, the choice of interpolating function of the base correlation at low strikes has little if negligible impact on the results.

We have computed the functions φ and $\tilde{\varphi}$ calibrated on the Itraxx index as of December 31th, 2004. They are represented on figure (4). We see on figure 2 that the function $\tilde{\varphi}$ is increasing and diverges for high confidence levels, which is coherent with intuition, and observed prices (prices on senior tranches are much above historical losses). Under the threshold of 51%, the function $\tilde{\varphi}$ is equal to 0 because 51% is the probability of having no defaults on the underlying pool of the ITraxx index. Above this threshold investors become risk averse, and the first plateau corresponds to having one default on the pool: the function is flat because the investors are indifferent to all of the confidence levels corresponding to one default on the underlying pool.

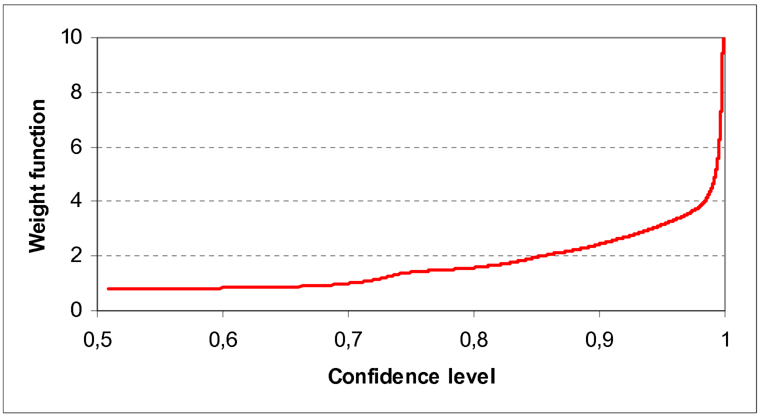

Figure 2: Weight function calibrated upon CDO market data as of December, 31th 2004.

The dynamic aspects of this calibration are interesting too because they open a field for modeling the dynamics of credit portfolios. We leave this issue for future work. We just notice that the impact of a stressed environment as the CDO market experienced in spring 2005 has the following impact on the $\tilde{\varphi}$ function (see figure(4):

Our goal in both following sections is to propose some applications of the concepts we have presented to the bank's risk management world.

5 APPLICATION TO A BANK PORTFOLIO'S VALUATION

In this section, we use the idea (assumption) that the pricing on the I-Traxx represents correctly the view of the market on the cost of risk for credit risk in general. The starting point of our idea is that the capital structure of a bank is a CDO with, on the assets side, a portfolio of bonds, loans and other types of facilities, and, on the liabilities side, the issuance of several quality notes, such as equity, subordinated debt, senior debt, super senior liabilities (deposits).

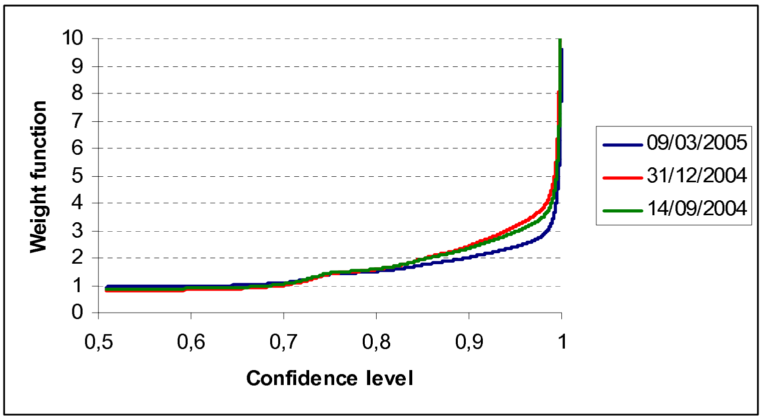

Figure 3: Weight functions as of dates 14/09/2004, 31/12/2004 and 09/03/2005.

Roughly speaking, a bank is a huge special purposed entity, whose cost of risk function could be calibrated upon market data.

However, the I-Traxx index and the bank's portfolios are different in terms of credit quality and concentrations. The I-Traxx index is made of investment grade CDS and is concentrated on large European corporate. On the other hand, a bank's portfolio, is often, on average, speculative grade, and diversified in terms of geography, products and types of counterparties. Obviously then, the function φ for both portfolios are going to be very different to each other. On the other hand, we can expect that the investors investing in the I-Traxx tranches and in the bank's tranches, are going to price them the same way for a given risk level. This is the reason why we are going to assume that the function $\tilde{\varphi}$ is the same for any credit portfolio.

This assumption has the following economical interpretation:

- Specific and systemic risk have the same cost of risk.
- The cost of risk is geographically homogeneous.
- The investors' pricing function (risk aversion) does not depend on the maturity of the risks they are holding (5 years for the I-Traxx and generally less for a bank's portfolio).

The first application of our methodology deals with loan pricing. The first step is to compute the bank's valuation function as a function of the total loss. As we know this function in the quantiles space (this is the universal $\tilde{\varphi}$ function), we can write:

$$\varphi_{Bank}(l) = \tilde{\varphi}\left(F_{Bank}(l)\right), \tag{15}$$

where the function F_{Bank} is the bank's historical loss distribution function. This function is directly available from Monte-Carlo simulations of the bank's portfolio. If we call $\mathbb{P}_{Bank}$ the historical probability measure of the bank's losses, we get the price of risk of the i-th individual transaction of the bank's portfolio as:

$$\mathbb{E}_{\mathbb{P}_{Bank}}\left[L_i \varphi_{Bank}(L)\right], \tag{16}$$

where L is the loss on the portfolio and L_i is the loss on the i-th operation.

When equation (16) is applied to future exposures, it leads to pricing an individual facility at a future date. The methodology described here can be applied to derive the bank's portfolio value distribution at any horizon just by running simulations under the historical probability measure $\mathbb{P}_{Bank}$, that is through the internal assessment methodology of the bank .

Phi ne bouge pas (ptf reconduit à l'identique)

As emphasized by Wang ([6]), in contrast with the expected shortfall contribution, this risk contribution has the advantage to take into account the whole loss distribution instead of the tails. It is generally much easier to compute loss distributions under the historical probabilities because the calibration of the parameters is more straightforward, especially for structured transactions. This issue is indeed of a first importance when double defaults are in the core of the risk of the transaction. The trick we use here is to make the simulations under the historical probabilities instead of a less reliable risk neutral probability measure; on the other hand, the valorization function is and is calibrated upon CDO market data, in a way that, this is our bet, is more reliable than estimating risk neutral parameters for the simulations.

Another application of this theory is the pricing of the bank's zero coupon spread. If the bank's default definition is that the loss on the credit portfolio is larger than a given threshold (the economic capital), we get the price of the bank's risky bond :

$$Bond = 1 - LGD\ \mathbb{E}_{\mathbb{P}_{Bank}}\left[\mathbb{1}_{\{L>K\}}\varphi_{Bank}(L)\right]. \tag{17}$$

We can for instance use equation (17) to calibrate the economic capital K of the bank from the bond market data instead of the rating aimed by the bank. Theoretically, if the hypothesis described in the beginning of the section are correct, our approach should be self-consistent and both definitions of the economic capital should be close to each other.

6 COMPUTING $\tilde{\varphi}$ AT THE BANK LEVEL

We propose in this section a method to construct $\tilde{\varphi}$ at the whole bank's portfolio level when the cost of risk has been calibrated on several sub-portfolios.

Banks have developed since several years internal models enabling them to measure their risk. Most of them are rating-based model and consist in simulating the life of their assets till they mature or default (buy-and-hold point of view, or Mark-to-Loss model).To put in a nutshell, a bank portfolio model is the aggregate of :

- a portfolio composition (with ratings, amortization profile),
- a dynamic structure (transition matrices),
- a dependance structure: risk factors, copulas.

Mark-to-Loss models are more easy to handle and to calibrate, which explains why they have been more popular. Recently, banks have began to value their loans portfolio by mark-to-marketing them. The mark-to-market of a loan portfolio is its market value, or equivalently (when the no arbitrage condition holds) its cost of hedging. Mark-to-market pricing of a loan portfolio very often implies a "new model" for the bank, and this additional model may not match -under "historical" assumption the results of the "former" model under mark-to-loss or historical assumptions. We advocate that a loss distribution produced through a mark-to-loss portfolio model can be easily converted into a mark-to-market loss distribution through $\tilde{\varphi}$ without resorting to an additional model.

We consider a bank whose portfolio consists in two sub-portfolios of loans in the same sector (for example: industrial sector in United States for the first sub-portfolio and mortgages in Germany for the second one). The dependance between both sub-portfolios is assessed through the bank's internal model that is able to perform for each α $Var_\alpha^{\mathbb{P}}(L_1)$, $Var_\alpha^{\mathbb{P}}(L_2)$, $Var_\alpha^{\mathbb{P}}(L_1{+}L_2)$, where L_1 and L_2 are the losses on sub-portfolios 1 and 2, and $\mathbb{P}$ is the probability measure used by the bank, that is, in other terms, its portfolio model.

The mark-to-market distribution of losses or equivalently the distribution of the price of risk on the total bank portfolio is the distribution of the $\varphi_{Bank}(L)L$ (where $L = L_1 + L_2$). First, we observe that the total price of risk on the bank's portfolio is

$$\begin{aligned} POR_{Bank} &= \int_0^{L_{1,Max}} \int_0^{L_{2,Max}} \varphi_{Bank}(l_1 + l_2)(l_1 + l_2)\mathbb{P}\left[L_1 \in dl_1,\ L_2 \in dl_2\right] \\ &= \int \int_{l_1,l_2} \varphi_1(l_1)\varphi_2(l_2) \cdots [COPULA] \end{aligned} \quad (18)$$

We construct φ_{Bank} in the following way:

$$\tilde{\varphi}_{Bank}(\alpha)VaR_\alpha^{\mathbb{P}}(L) = \varphi_1(VaRC_\alpha^{\mathbb{P}}(L_1))VaRC_\alpha^{\mathbb{P}}(L_1) + \varphi_2(VaRC_\alpha^{\mathbb{P}}(L_2))VaRC_\alpha^{\mathbb{P}}(L_2), \quad (19)$$

where $VaRC_\alpha^{\mathbb{P}}(L_1)$, $VaRC_\alpha^{\mathbb{P}}(L_2)$ stand for the risk contributions of L_1 and L_2.

NOTE:
Is probably correct only when $\tilde{\varphi}$ has been defined as a ratio of VaR.
END NOTE

7 APPLICATION TO VALUE BASED MANAGEMENT

TO BE COMPLETED

In this section, we apply our approach to the bank's value based management, and in particular to measuring a differentiated cost of capital for each business line. It is possible to extract from market data the average cost of capital of the whole bank. Indeed, the beta measured from stock price returns is linked to the return expected by investors trough the CAPM relationship. However, deriving the cost of capital at the business line level requires knowledge on the bank's portfolio at least at this level. Since we now have simulation tools that lead to the price of risk, the expected loss and the contribution to economic capital, we can define the cost of risk of the business line number i by:

$$k_i = \frac{\mathbb{E}_{\mathbb{P}_{Bank}}\left[L_i \varphi_{Bank}(L)\right] - \mathbb{E}_{\mathbb{P}_{Bank}}\left[L_i\right]}{EC_i}, \tag{20}$$

where is L_i is the loss on the transaction i, L is the loss on the portfolio and EC_i is the risk contribution of the i-th transaction. Deriving a cost of capital for each business line is a tricky exercise. Work has been proposed by Wilson ([see ref ?]) who proposes to adapt a CAPM model on the assets of the firm which are subject to different sources of risk. Wilson computes hurdle rates for specific business lines. However, as underlined by de Servigny [], computing a cost of capital may lead to undesirable arbitrages inside a bank where two business lines could enter the same transaction with differentiated costs of funds.

Our approach enables us to circumvent both difficulties. Each transaction has its own cost of funds inside the whole portfolio of the bank, and this average cost of funds is given by :

$$k_i = \frac{\mathbb{E}^{\mathbb{P}_{Bank}}\left[L_i \varphi_{Bank}(L)\right] - \mathbb{E}^{\mathbb{P}_{Bank}}\left[L_i\right]}{Exposure} \tag{21}$$

The cost of Equity c_i for this operation is then derived thanks to the following equality:

$$k_i \ . \ Exposure \ = c_i \ . \ EC_i + Spread_{Bank} \ . \ \left[Exposure \ - \ EC_i\right], \tag{22}$$

where EC_i is the risk contribution that has been computed through the bank's internal model. This risk contribution can be computed as a Credit VaR, or as an Expected Shortfall.

[KEEP THAT ?]

Companies listed in the ITRAXX index represent several sectors that can constitute indexes. These sectors are for Europe Autos, Consumers, Energy, Financials, Industrials and TMT. Then sum of the cost of risk on all sub-portfolios (Autos,) should be equal on the cost of risk of the aggregated portfolio. To simplify, let us imagine we only have two portflolios. P_1 and P_2. We want to compute the cost of risk on the portfolio $x_1 P_1 + x_2 P_2$. When the total price of risk on the whole portfolio is equal to the sum of the total price of risk for both portfolios, we obtain:

$$POR(x_1P_1 + x_2P_2) = \int_0^1 \tilde{\varphi}(\alpha)VaR_\alpha(x_1P_1 + x_2P_2)d\alpha \quad (23)$$

$$= x_1 \int_0^1 \tilde{\varphi}(\alpha)VaR_\alpha(P_1)d\alpha + x_2 \int_0^1 \tilde{\varphi}(\alpha)VaR_\alpha(P_2)d\alpha \quad (24)$$

and we also have the following constraint on the VaRs:

$$\int_0^1 VaR_\alpha(x_1P_1 + x_2P_2)d\alpha = x_1 \int_0^1 VaR_\alpha(P_1)d\alpha + x_2 \int_0^1 VaR_\alpha(P_2)d\alpha \quad (25)$$

However, we already have: in general and and

Then, we have :

The price of risk is non additive, and we can check on the ITRAXX that the sum of the spread on all the tranches is not equal to the sum of spreads on individual names, except if the underlying pricing model is a one-factor model. This gives arguments for a cautious identification of sources of risk under historical measure. When sources of risk are multiple, the price of risk on two portfolios is not equal to the sum of the prices of risk on individual portfolios. Here, because of the plateau on Phi tilde for low alpha, we can expect that the total price of risk on portfolio P1+P2 is lower than the total price of risk for portfolio 1 and portfolio 2. However, should the whole portfolio be tranched, spreads for first tranches would be increased. [Detail more that: gives more power to high quantiles .] Could help to "optimize" structuration. This would show that Cost of Risk is a sub-additive measure and is not additive.

But and more generally, since VaR is not a sub-additive risk measure, we cannot predict the sign of This implies restricted forms for Phi tilde. Somme des spreads de CDO ¡¿ spread panier

Ide sous jacente : si on fait du pricing base d'une mesure de performance qui fait intervenir une mesure de risque sous additive, sous contrainte d'un critre de rentabilit minimal, on n'est pas conforme l'arbitrage.

Le cot du risque est la seule mesure cohrente qui parvienne rconcilier le tout (tre prudent : on ne prend pas le cot du risque sur la tranche [0,100

, which implies for traditional sub-additive risk measures:

[END KEEP THAT ?]

8 CONCLUSION

We have calibrated on the CDOs market the universal probability change from historical to risk neutral credit loss distributions. We have assumed that this Radon-Nikodym density expressed in the quantiles space was universal, and did not depend on the underlying portfolio, but only on the confidence threshold that we consider. We have discussed in detail the economical interpretation of this assumption, and we argue that it is relevant in a first order approach.

We applied this approach to the valuation of the loans (vanilla and structured) of a bank's portfolio, and we provided an answer to the issue of valuing the cost of capital per branch. The advantage of our approach is three-fold: first the calibration is much easier and reliable than calibrating a bank's portfolio in a risk neutral universe. Second it bridges in a conceptual way the market investors' views on structured credits to the world of banking corporation; this leads to a new understanding on how optimal management of a bank's balance-sheet is possible. Third, we have made a first step in deriving the dynamics of the structured credit investors' risk aversion, which seemed to be a fundamental element in the spring 2005 correlation crisis. We leave this last issue for future work.

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