Dealing with seller's risk

The risk of trade receivables securitisations comes from both the pool of assets and the seller of the assets. ***Vivien Brunel*** *develops a model for securitisation exposures that deals with both risks, and analyses in detail the interplay between debtors' risk and seller's risk*

In contrast with collateralised debt obligations (CDOs), which are now considered as vanilla products, asset securitisations exhibit a large variety of structures and underlying assets. On top of that, securitising assets induces another type of risk, namely seller's risk, which is a crucial issue in the rating agencies' criteria, especially for short-term assets such as trade receivables (Standard & Poor's, 1999). Debtors' risk and seller's risk are interdependent variables and, as far as we know, the interplay between them has never been explored in a quantitative perspective.

Trade receivables transactions are gathered in a conduit that is funded by the issuance of asset-backed commercial paper (ABCP). For each transaction, the credit quality of the pool of assets is enhanced by overcollateralisation and by driving the transaction into early amortisation when some triggers are hit. The role of the seller is to transfer the cashflows coming from the underlying assets to the conduit; if the seller defaults in the meantime, the conduit suffers some losses (commingling losses), even if the pool of assets is wealthy. Other losses may occur after the seller's default when a back-up servicer captures the cashflows of the assets because of the notification delays and of the reduced efficiency in the collections. Finally, losses may also occur when the seller makes a rebate or issues credit notes to its debtors and recovers only part of the expected collections; this is called dilutions. As long as the seller survives, he pays for the dilutions and the conduit does not have to support them, but this is no longer the case when he defaults.

The structure of the conduit is as follows:

- The seller holds the first-loss risk through overcollateralisation and several reserves.
- The ABCP investors fund the rest of the assets. They support only losses coming from the pool of assets.
- The originating bank offers the ABCP investors a first-loss protection through the issuance of a letter of credit. This exposure is a mezzanine exposure at the conduit level (mutualised over all the transactions); it is similar to a CDO squared exposure.
- The originating bank provides each transaction with a liquidity facility, which is a senior exposure for both seller's risk and debtors' risk. From a risk perspective, the ABCP investors hold a negligible part of the risk (the ABCP are rated A1+/P1). It is thus relevant to approximate the originating bank's exposure by a senior exposure on both debtors' risk and seller's risk (see figure 1).

Modelling the economics of securitisation transactions is of great interest for a structured products business line. Such a model is at the same time a front-office tool for optimally structuring transactions and a middle-office tool for dynamically managing risk. Moreover, in the Basel II framework, internal ratings-based banks have incentives to use internal models for calculating regulatory requirements on each exposure in the case of ABCP transactions. Developing such a model is a matter of reputation and of comparative advantage for banks.

Many models focus on credit risk tranching and CDO pricing (Bluhm, Overbeck & Wagner, 2002), but very few of them deal with asset securitisation. Vasicek's model (1991) has launched a whole family of analytical approaches for credit portfolios and Pykhtin & Dev (2002) have calculated economic capital within Vasicek's model. The Basel Committee for Banking Supervision has retained a methodology for securitisation (BIS, 2004) based on quantitative probabilistic methods (Gordy & Jones, 2003), but it does not actually include all the specific issues of asset securitisation, such as seller's risk or excess spread protection. Moody's Investors Service's (2000) lognormal model for asset-backed securities is one of the only economic-based models dealing specifically with asset securitisation and there in no doubt that this will constitute a very creative area for future research.

In this article, we show that quantitative methods are relevant for securitisation issues and lead to a deep understanding of complex phenomena. The article is organised as follows. First, we develop a simple analytical model for trade receivables transactions that includes seller's risk. We then obtain a closed-form formula for the economic capital of a securitisation exposure. Finally, we provide more details on the interplay between debtors' risk and seller's risk.

Simplified model for trade receivables: seller's risk

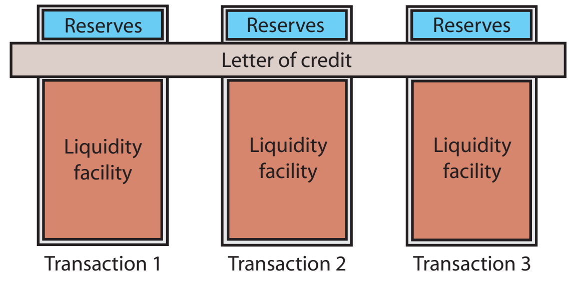
Here, we present a one-factor and one-period credit portfolio model including seller's risk. We call L the total loss supported by the structure. Losses come from defaults on the underlying pool of assets and/or from the default of the seller. The total loss over the time period writes as the sum of two terms:

$$L = \mu L_D + \left(1 - L_D\right) \times LGD \times X_C \tag{1}$$

The notations are provided in table A. In the second term, we notice that the exposure at risk when the seller defaults is the wealthy part of the pool of assets. The Basel Committee introduced a similar consideration for analysing the risk of overlapping exposures (BIS, 2004, paragraph 581), in order to prevent double counting of the risks and losses. We estimate the recovery μ on the defaulted assets from historical data, but the LGD parameter is more difficult to obtain because it summarises all the monetary consequences of the seller's default. We address this issue at the end of the next section.

In our model, we assume that the defaults of the seller and of the underlying assets are dependent random variables with a normal copula with correlation parameter ρ between them. To this end, we introduce a standard normal systemic risk factor Y common to both variables. For the portfolio of assets, we assume that the delinquencies write:

1 Exposures of the originating bank on the conduit: mutualised letter of credit at the conduit level and liquidity facilities at the transaction level



A. Notation and parameter definitions

L_D	Amount in default on the pool of assets (delinquencies)
μ	Loss given default on the underlying assets
X_c	Binomial variable equal to one if the seller defaults and zero if he survives
$p_c = N(s_c)$	Seller's default probability. The function $N(\cdot)$ is the cumulative standard normal distribution function
LGD	Monetary consequences of the seller's default
Y	Standard normal risk factor driving the economy of the structure

$$L_D = F(aY + b) \tag{2}$$

where $a < 0$ and b are parameters depending on the pool characteristics, and the increasing function F is related to the distribution function of the amount in default L_D. When $F(\cdot) = \exp(\cdot)$, the distribution function of L_D is lognormal. This is a common model in the field of cashflow securitisation modelling and has been popularised by Moody's (2000). When $F(\cdot) = N(\cdot)$, where $N(\cdot)$ is the cumulative standard normal distribution, the pool default distribution is Vasicek's law (1991). This is the limit law of a pool of loans in the homogeneous and infinitely granular limit. In the lognormal case, the parameters a and b are easily related to the first two moments of the distribution, and, in the case of Vasicek's law, they are related to the expected default rate and to the pair-wise asset correlation. Table B summarises these results.

We assume that the default of the seller is described by a binomial random variable:

$$X_C = 1_{\left\{\sqrt{\rho}Y + \sqrt{1-\rho}\varepsilon_C < s_C\right\}} \tag{3}$$

where ε_C is a standard normal variable independent of Y. The economic interpretation is that the random variable driving the seller's default is made of a systemic part Y (shared in common with the rest of the structure) and of an idiosyncratic part ε_C. As a first step in understanding better the interplay between debtors' risk and seller's risk, we can compute the pool loss distribution conditional on the default of the seller. We have:

$$P\left(Y < y \middle| \sqrt{\rho}Y + \sqrt{1-\rho}\varepsilon_C < s_C\right) = \frac{N_2\left(y, s_C, \sqrt{\rho}\right)}{N\left(s_C\right)} \tag{4}$$

where $N_2(\cdot\,,\cdot\,,\cdot)$ is the bivariate cumulative normal distribution function. We derive an analogous formula for the distribution function of the systemic factor Y conditional on the seller's survival. In both cases, we are also able to calculate the moments of these conditional laws, and it turns out that both their skewness and the excess kurtosis are very close to zero. In what follows, we state that the conditional law of the systemic factor Y is approximately normal. This assumption is supported by table C.

B. Lognormal and Vasicek laws

Model	L_D	Expected value	Std dev/asset corr
Lognormal	e^{aY+b}	$e^{b-a^2/2}$	$e^{b-a^2/2}\sqrt{e^{a^2}-1}$
Vasicek	$N(aY+b)$	$N(b/(\sqrt{1+a^2})$	$a^2/(1+a^2)$

C. Skewness and excess kurtosis of the systemic factor distribution function conditional on the seller's default

s	*N*(*s*) (bp)	ρ	Skewness	*Xs* kurtosis
–1	1,586.6	10%	–4.19E-03	9.30E-04
–2	227.5	10%	–2.16E-03	4.73E-04
–3	13.5	10%	–1.15E-03	2.30E-04
–4	0.3	10%	–6.56E-02	1.16E-04
–1	1,586.6	20%	–1.36E-02	4.49E-03
–2	227.5	20%	–7.11E-03	2.33E-03
–3	13.5	20%	–3.83E-03	1.14E-03
–4	0.3	20%	–2.19E-03	5.81E-04
–1	1,586.6	30%	–2.90E-02	1.23E-02
–2	227.5	30%	–1.55E-02	6.58E-03
–3	13.5	30%	–8.44E-03	3.28E-03
–4	0.3	30%	–4.86E-03	1.68E-03
–1	1,586.6	40%	–5.28E-02	2.74E-02
–2	227.5	40%	–2.89E-02	1.51E-02
–3	13.5	40%	–1.60E-02	7.67E-03
–4	0.3	40%	–9.28E-03	3.97E-03

From equation (4), we derive the expected value and variance of the systemic factor conditional on the seller's default:

$$\begin{aligned} m_D &= E\left[Y \middle| X_S = 1\right] = -\sqrt{\rho}\frac{n\left(s_C\right)}{N\left(s_C\right)} \\ v_D &= \operatorname{var}\left[Y \middle| X_S = 1\right] = 1 - \rho s_C \frac{n\left(s_C\right)}{N\left(s_C\right)} - \rho\frac{n^2\left(s_C\right)}{N^2\left(s_C\right)} \end{aligned} \tag{5}$$

where the function $n(\cdot)$ is the standard normal density function. Similar expressions hold for the mean and variance of the systemic factor Y conditional on the seller's survival:

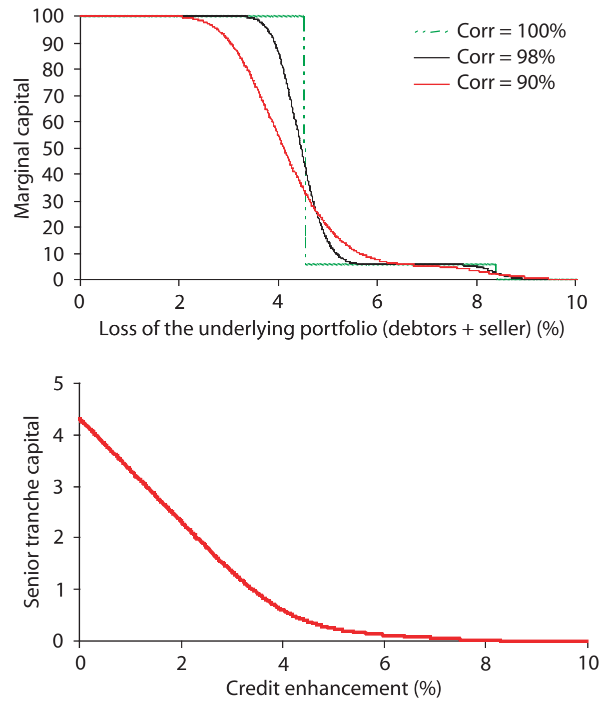
$$\begin{aligned} m_{\bar{D}} &= E\left[Y \middle| X_S = 0\right] = \sqrt{\rho}\frac{n\left(s_C\right)}{1 - N\left(s_C\right)} \\ v_{\bar{D}} &= \operatorname{var}\left[Y \middle| X_S = 0\right] = 1 + \rho s_C \frac{n\left(s_C\right)}{1 - N\left(s_C\right)} - \rho\frac{n^2\left(s_C\right)}{\left(1 - N\left(s_C\right)\right)^2} \end{aligned} \tag{6}$$

These results are exact. The only assumption we make in what follows is that the law of the systemic factor conditional on the default (respectively, survival) of the seller is a normal law with parameters m_D and v_D (respectively, $m_{\bar{D}}$ and $v_{\bar{D}}$).

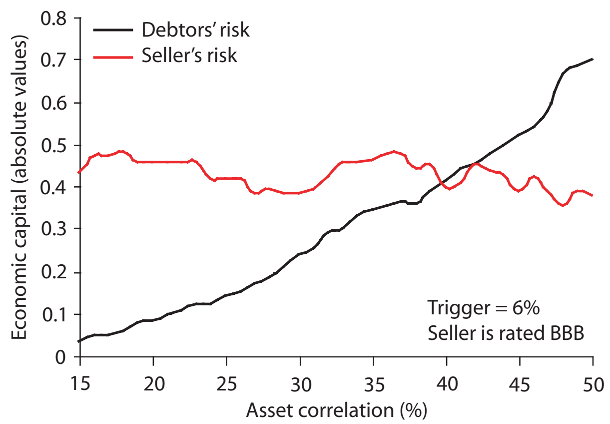
Capital requirement formula

We introduce the notion of an infinitely thin tranche (ITT) for securitisation exposures. An ITT with attachment point l is a mezzanine tranche of risk with attachment point l and detachment point $l + dl$. In the limit of vanishing thickness of the tranche ($dl \to 0$), the loss on the ITT is the binomial random variable $1_{\{L \geq l\}}$. Following Pykhtin & Dev (2002), we make the

2 Capital requirement for ITT tranches and senior tranches (factor correlation 90%)



3 Split between debtors' risk and seller's risk of the economic capital in absolute values as a function of the asset correlation



following assumptions:

- A securitisation exposure is a part of a large diversified bank's portfolio. The bank's portfolio is fine-grained and the securitisation exposure has a negligible size compared with it.
- A systemic factor Z drives the bank's portfolio losses.
- The dependence of the pool systemic factor Y with the systemic factor Z is linear: $Y = \sqrt{\rho_Y}Z + \sqrt{1-\rho_Y}\,\varepsilon$. The variable ε is standard normal and independent of Z.

The economic capital of a marginal exposure is equal to its expected loss conditional that the bank's portfolio loss is equal to its default threshold (Gourieroux, Laurent & Scaillet, 2000). As the marginal exposure is assumed to be infinitesimal, the default threshold of the bank's portfolio (including the marginal securitisation exposure) is defined by the condition that the systemic factor Z is equal to its q-percentile; this percentile defines the economic capital associated with the bank's portfolio. The marginal contribution to the economic capital of an ITT with attachment point located at l is:

$$Cb(l) = E\left(1_{\{L\geq l\}} \middle| Z = N^{-1}(1-q)\right) = P\left(L \geq l \middle| Z = N^{-1}(1-q)\right) \quad (7)$$

Conditional on the systemic factor being equal to its $(1-q)$ percentile, equation (3) leads to the seller's default condition:

$$\sqrt{\rho^q}\,\varepsilon + \sqrt{1-\rho^q}\,\varepsilon_C < s_C^q \quad (8)$$

where:

$$s_C^q = \frac{s_C - \sqrt{\rho\rho_Y}\,N^{-1}(1-q)}{\sqrt{1-\rho\rho_Y}} \qquad \rho^q = \frac{\rho(1-\rho_Y)}{1-\rho\rho_Y} \quad (9)$$

Conditional on the value of the systemic factor Z being equal to its $(1-q)$ percentile, we write the default on the pool of assets as a function of the random variable ε:

$$L_D^q = F\left(a^q\varepsilon + b^q\right) \quad (10)$$

with:

$$a^q = a\sqrt{1-\rho_Y} \quad \text{and} \quad b^q = b + a\sqrt{\rho_Y}\,N^{-1}(1-q)$$

By conditioning upon the two possible states of the seller (survival and default), the formula of the contribution to economic capital given by equation (7) becomes:

$$Cb(l) = P\left[\mu L_D^q \geq l \middle| X_C = 0\right] P\left[X_C = 0\right] + P\left[\mu L_D^q + LGD\left(1-L_D^q\right) \geq l \middle| X_C = 1\right] P\left[X_C = 1\right]$$

After plugging the results of equations (5), (6) and (9), we obtain:

$$\begin{aligned} Cb(l) = {} & \left[1 - N\left(s_C^q\right)\right] N\left(\frac{F^{-1}(l/\mu) - b_D^q}{a_D^q}\right) 1_{\{l<\mu\}} \\ & + N\left(s_C^q\right)\Bigg[1_{\{l<LGD\}} + \left(1_{\{l>LGD\}\cap\{\mu>LGD\}} - 1_{\{l<LGD\}\cap\{\mu<LGD\}}\right) \\ & \times N\left(\frac{F^{-1}\left(\frac{l-LGD}{\mu-LGD}\right) - b_D^q}{a_D^q}\right)\Bigg] \end{aligned} \quad (11)$$

where:

$$a^q_{D,\bar{D}} = a^q v^q_{D,\bar{D}} \qquad b^q_{D,\bar{D}} = b^q + a^q m^q_{D,\bar{D}} \quad (12)$$

The parameters $m^q_{D,\bar{D}}$ and $v^q_{D,\bar{D}}$ are obtained from equations (5) and (6) by substituting s_C and ρ by s_C^q and ρ^q respectively. When the pool loss follows Vasicek's law and in the limit where there is no seller's risk ($s_C^q \to -\infty$), we recover the result of Pykhtin & Dev (2002). If the seller is likely to default but $LGD = 0$, we do not recover exactly the result of Pykhtin & Dev because of the assumption of equations (5) and (6). However, numerically, the distribution functions are very close to each other. Having derived the capital for any ITT, we are able to determine the capital of any securitisation exposure with attachment point T_1 and a detachment point. We decompose this tranche into an infinite number of ITTs, and the capital of the total exposure is the sum of the capital required for each ITT:

$$K(T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} dlCb(l) \tag{13}$$

We can check with numerical examples that the assumption of the normality of the systemic factor Y is relevant and leads to very accurate results, for instance by comparing the results of the analytic model with Monte Carlo simulations. Let's take an example. We assume that the seller's default probability is 0.256% (close to a BBB rating), and that seller's risk amplitude is $LGD = 5\%$. The default rate of the pool follows Vasicek's law with average default rate 2% and internal correlation 20%; the other correlation parameter $\rho = 20\%$, the pool loss given default is $\mu = 20\%$, the factor correlation ρ_Y is in the range 90–100%, and the confidence interval for the bank's portfolio is $q = 99.9\%$. The top graph in figure 2 shows the function $K(T_1, T_2)$ in the limit $T_1 = T_2$ (IIT case); the bottom graph corresponds to the economic capital of the senior tranche ($T_2 = 100\%$) as a function of the subordination.

In the preceding example, the LGD parameter summarises the monetary consequences of the seller's default. The Basel Committee associate seller's risk with dilution risk, meaning that the LGD parameter is equal to the average rebate that the seller makes to the debtors on the receivables. However, from an economic viewpoint, there are other risks linked to the seller. Indeed, when the seller defaults, the main risk for the transaction is that past collections are not yet transferred to the conduit (commingling risk), and that future collections are imperfectly transferred. This risk can be mitigated by increasing the collections transfer frequency or by choosing an efficient back-up servicer. We emphasise here the non-zero value of the LGD variable, even if the transaction is bankruptcy-remote.

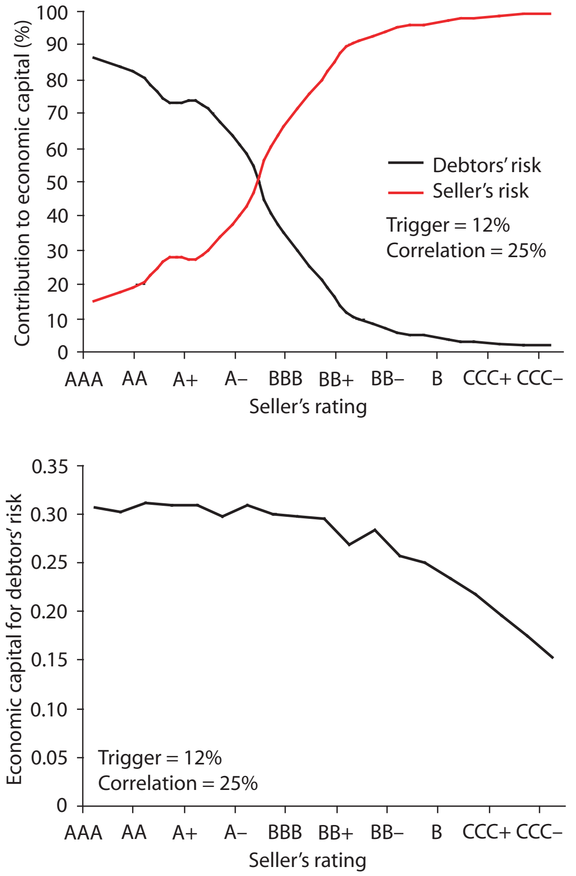
More on the interplay between debtors' risk and seller's risk

The weakness of the analytical model developed in the previous sections is that it is a static one-period model. We have developed a more realistic Monte Carlo model for securitisation exposures that goes far beyond the analytical model of equations (11) and (12), taking dynamically into account the asset replenishment and the triggers of the transaction. This section is devoted to analysis of the interplay between debtors' risk and seller's risk within the simulation model. In the following examples, the base case we consider is a BBB rated seller with loss given default equal to 5%, a default rate of 2% on the pool of debtors, an internal asset correlation equal to 25%, and a default rate trigger threshold equal to 6%. We study the absolute and relative sensitivity of the debtors' and seller's contributions to the economic capital relative to the asset correlation, seller's rating and default rate trigger, respectively. The securitisation exposure in the scope of our study is the tranche 10–100%.

At first sight, the independence between debtors' risk and seller's risk is well established by the argument that the first is related to the underlying pool of assets, whereas the second is considered as an operational risk, involving the seller. However, the pool of assets and the seller are correlated together, inducing a positive correlation between the underlying pool loss and the seller's default. Indeed, the financial wealth of the seller is likely to be correlated to its primary business. Figure 3 shows the impact of the internal correlation on the contributions of both risks in absolute values.

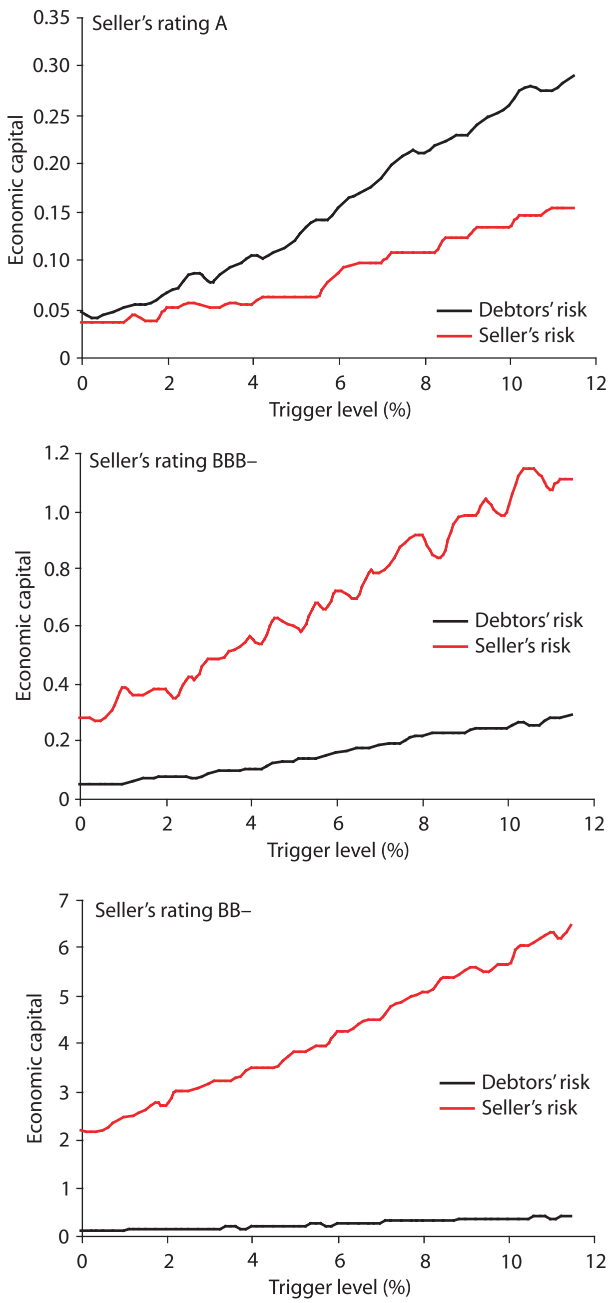
There is a clear increase in the debtors' risk contribution with the internal asset correlation. This is because the pool loss volatility increases with the asset correlation, and then the pool becomes riskier when the internal asset correlation increases. On the other hand, the absolute contribution of seller's risk to the economic capital remains almost stable. Indeed, the sensitivity of this contribution to asset correlation is of second order since it comes only from the exposure at default equal to $1 - L_D$, as explained in equation (1). The correlation between seller's risk and debtors' risk is mitigated by the fact that the higher the debtors' risk is, the lower seller's risk is.

4 Split between debtors' risk and seller's risk of the economic capital in relative values (top) and absolute values (bottom) as a function of the seller's rating



A second driver of the interplay is the seller's rating. When the seller's quality deteriorates, the contribution of seller's risk increases as expected. The relative contribution of debtors' risk decreases at the same time, exhibiting a regime switch phenomenon. Roughly speaking, when the seller is investment grade, large losses on the pool occur and make the transaction enter early amortisation before the seller defaults. Conversely, when the seller is speculative grade, large losses on the pool are less likely to occur before the seller defaults, in spite of the correlation between the seller and the debtors. This explains the 'X' curves of the left-hand graph in figure 4. This also explains the shape observed on the right-hand graph. On this graph, we see clearly the regime switch between the region where the seller is investment grade and has no impact on the debtors' risk, and the region where the seller is speculative grade and has an impact on the absolute value of the economic capital contribution of debtors' risk.

5 Debtors' risk and seller's risk contributions as a function of the trigger level for A, BBB– and BB– seller's rating



Note: delinquencies on the pool of debtors are equal to 2% on average

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Finally, a last driver of the interplay comes from the dynamic aspects of the transactions. The absolute value of the contributions effectively depends on the value of the trigger. We see that economic capital is an increasing function of the trigger's value because economic capital gives credit to large losses whereas a low default trigger limits high losses. This illustrates the role of the trigger as a risk mitigant and credit enhancement mechanism, but also that the fine-tuning of the trigger is a key driver of the risk profile of the transaction. In figure 5, we show that both the debtors' risk economic capital and the seller's risk economic capital are increasing functions of the trigger level.

As in the case of the seller's rating sensitivity, the sensitivity on the trigger level is driven by the dynamics of the transactions, and, in particular, on the timing of large losses on the pool of assets relative to the seller's default. Another explanation of this sensitivity is the economic situation (summarised in the systemic factor Y) conditional on hitting a trigger. If the trigger is low, the deal will be likely to enter early amortisation, even in an unstressed economic context. In this situation, the occurrence of losses on the senior tranche is very unlikely since senior tranches are mainly sensitive to systemic risk. On the other hand, if the trigger is difficult to reach, the trigger will be hit in a stressed systemic context and losses on the senior tranches will be more likely to occur.

Conclusion

In the field of asset securitisation, the range of counterparties is often very large, including the debtors, the seller, and also eventually insurers and other third parties. Either directly or indirectly, the performance of the transaction and the risk supported by the different investors depend on the health of all these counterparties.

In this article, we have developed a model for securitisation transactions including both debtors' risk and seller's risk. Using a realistic Monte Carlo model for trade receivables securitisations, we show that debtors' risk and seller's risk are highly interdependent variables. This model is relevant for a business line to structure optimised transactions, for risk management and performance measurement purposes. Going beyond the classical cashflow models and modelling the underlying assets and the clauses of the transactions is an open field of research. The recent convergence of regulatory capital and economic capital, based on quantitative probabilistic methods, proves that this research is very topical and timely. ■

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