USING CREDIT DERIVATIVES?

QUANTS N° 40

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YOUR WORLD OF FINANCIAL SERVICES
Overview

Recent awareness of the need to manage credit risks has spurred the development of credit derivatives. These products allow financial institutions to reduce or eliminate their credit risk. A broad range of credit derivatives is now available, each structured to meet a specific aim. But the market is struggling at present. The problems stem from the nature of the underlying interest, the legal definition of credit events, and the complexity of the modelling process. With the reform of the Cooke ratio, the question of credit has aroused considerable attention. The aim of this issue of Quants is to contribute to the debate. It examines the key aspects of credit derivatives by examining the diversity of the products available. And, by analysing the market's shortcomings, to understand the role played by pricing models.

November 2001

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INTRODUCTION

The recent expansion of the market in credit derivatives is a major highlight in the world of investment banking. The amounts outstanding in these instruments rose from $200 billion in 1997 to $800 billion in 2000, a four-fold increase in as many years. As their name implies, credit derivatives offer bankers a new way of operating in the credit market, which has grown as a result of three main factors. First, the deregulation of European markets and the advent of monetary union has resulted in greater liquidity and more competition, creating a truly homogenous European credit market. Second, given the low level of nominal interest rates, final investors are willing to take on more credit risk to boost returns. And in the aftermath of the recent crises in Asia and Russia, the need to hedge and evaluate these investments has become patently clear. Third, the regulatory authorities are set to accept the use of internal models for measuring risk. This will enable banks to better identify and measure credit risk and therefore manage it more effectively.\(^3\)

The credit derivative market is struggling to overcome a number of difficulties arising from the legal definition of credit events, the nature of the underlying and the complexity of the modelling process.

One of the biggest hurdles to the development of credit derivatives is their legal and operational complexity. In the equity markets, for example, the exercise of an option is not subject to challenge because the price of the underlying is set for each transaction. This is certainly not the case for options on credit events insofar as there is a conflict of interest between the two parties to the credit derivative contract. So for each contract, it is important to precisely define the credit events that will generate cashflows, as well as the method of payment. These questions are addressed in the first section of this issue of Quants.

Another hindrance stems from the nature of the underlying interest. Over the past few decades, the markets in equities, bonds and currencies have become truly liquid. In turn, organised markets in derivatives have also become liquid. But the market in risky debt has not reached the same level of liquidity, and this affects hedging strategies. Furthermore, there are purely artificial credit derivatives whose payoffs depend, say, on corporate events, even though the company in question has no bond debt. All this goes to show that the underlyings for credit derivatives are more complex than for shares, bonds and foreign exchange. What is more, they are sometimes impervious to traditional methods of analysis, notably quantitative techniques.

A number of mathematical methods have been devised for modelling credit risk, but the results depend heavily either on the basic assumptions regarding default

\(^3\) See Quants n° 38 for a more detailed discussion of this issue.
probability or on the fact that default risk alone explains the spread between the corporate debt rate and a benchmark rate such as the T-bond yield or swap rate. While these assumptions are practical for modelling purposes, they are way out of line with reality. Such models cannot be fully effective unless the market in question is sufficiently efficient - as is the case with the equity, bond and forex markets.

In Part One of this issue, we present in detail the different types of credit derivatives and their financial mechanisms, depending on whether they are intended for hedging or speculation. In Part Two, we describe structures such as collateralised bond obligations (CBOs), collateralised loan obligations (CLOs) and synthetic securitisations, which are based on credit derivatives. In particular, we point out the advantages for financial institutions of structuring these products and we explain how they can be used to optimise capital allocation. In the third and final part, we discuss the latest innovations in credit risk modelling. We show in particular how the uncertainty stemming from the nature of the underlying can be taken into account in a modelling process.

PART ONE

THE CREDIT DERIVATIVES MARKET

1.1 Introduction to credit risk

In practice, there is one crucial difference between a bond issued by a company and one issued by a government, namely the level of solvency. An investor who lends to a private enterprise considers that the risk of default is greater than if he were lending to a government. So he will demand a higher rate of interest to compensate for that risk. The credit differential - or spread - between these two issuers will thus depend on the probability of either one defaulting during the life of the bond. It should be noted that a spread is also defined as the difference between the corporate debt rate and the swap rate in the bank financing market. Furthermore, by purchasing a risky (or "defaultable") bond and immunising the portfolio against interest rate risk, the investor is then exposed solely to the issuer's credit risk via fluctuations in its spread. The spread thus becomes a financial instrument in its own right, in almost exactly the same way as an interest rate.

The value of the spread depends on the company's level of default risk and can vary following a shift in investor perception of that risk. A finer-grained approach consists in analysing the risk of a change in the rating assigned to a debt issue. These ratings are issued by independent agencies and have a major impact on the market value of credit spreads.
The term "credit risk" also applies to the counterparty risk embodied in a derivatives contract, notably for credit derivatives.

I.2 Types of credit derivative

We have seen that spreads and ratings are bellwethers of credit risk and that credit risk can be traded in very much the same way as interest rate risk. The development of credit derivatives is therefore a natural extension of this situation. Credit derivatives can be divided into three main categories depending on their mechanisms and on the aims of their users:

- credit derivatives based on default risk
- credit spread derivatives, based on differences in creditworthiness
- products that synthetically replicate the performance of the underlying.

The first category includes credit default swaps (CDSs) and credit linked notes (CLNs). The second includes credit spread options (CSOs) and spread forwards. And the third group comprises total return swaps (TRSs) and total return linked notes. We will now take a detailed look at CDSs, CLNs, CSOs and TRSs.

• Credit default swaps

CDSs are used to transfer credit risk from one party to another without exchanging the securities that underlie the contract. The purchaser of a CDS is buying protection against credit events affecting the reference security.

No cashflows are exchanged at the outset of the transaction. The fixed leg of the swap consists of periodic payments of a fixed margin (premium). The floating leg is a payment conditional on a default event: it is zero if no default occurs and is equal to the change in the price of the reference security in the event of default. The diagram in Exhibit A shows the cashflows generated by a CDS.

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Exhibit A – Credit Default Swap

Bank A is a protection buyer. It pays spread $x$, known as a premium, at regular intervals. Bank B is a protection seller: if a credit event occurs on reference issuer C, Bank B pays pre-arranged cashflows to Bank A.

\[
\begin{align*}
A & \quad \text{Premium } x \text{ bp} \\
& \quad \text{=0 if no default} \\
& \quad \text{≠ 0 if default} \\
C & \\
B
\end{align*}
\]

Note that entity A is not required to hold the reference security of Company C. Indeed, in some cases, Company C has not even issued bonds.

The premium depends on Company C’s default probability and the expected recovery rate of its debt (see Part Three for further details); it is adjusted so that the value of the CDS is zero at the outset of the transaction.

As regards the two parties’ exposure, Bank A is exposed to Bank B’s counterparty risk via the floating leg of the swap. Bank B is exposed to the credit risk of Company C – which is the whole point of the transaction.

In practice, a CDS is a tool for hedging spread risk. When the spread of C’s issue changes, the value of the CDS changes accordingly.

To understand how a CDS works, consider the following example. Company C want to borrow €100 million from Bank A and can afford to pay a spread of 25 bp over a benchmark rate, say Euribor. The deal interests Bank A, which can fund at a spread of -10 bp but it can lend only €50 million because of its line of credit on C. However, it wants to do the whole deal with Company C so as to maintain its commercial relationship. Bank B could take on one-half of the transaction, but its cost of funds, namely 10 bp, would yield a margin of just 15 bp, which it considers inadequate. For A and B, the solution consists in negotiating a CDS on Company C. Bank A lends Company C €100 million but transfers the credit risk on one-half of the transaction to Bank B via the fixed leg of the CDS with a 25 bp spread. In return, Bank B offers Bank A a guarantee on a nominal amount of €50 million in the event that Company C defaults.

The outcome of the deal is as follows. Bank A earns a margin of 35 bp on a nominal amount of €50 million and 10 bp on a nominal of €50 million. Also, it strengthens its relationship with Company C by taking on the entire deal. Bank B generates a 25 bp
margin on a nominal amount of €50 million and is able to diversify its loan portfolio. Moreover, thanks to the deal, it also has an opportunity for regulatory capital arbitrage.

- **Credit linked notes**

From a historical perspective, the CLN was the first-ever credit derivative. Unlike CDSs, CLNs require the investor to fund the credit risk position by paying the nominal amount at the initial date. CLNs are debt securities with an embedded option on the default of the reference borrower. They allow bond investors to diversify their portfolios by acquiring the paper of a borrower other than the issuer of the note. For the issuer, a CLN is a way of disengaging itself from the risk and the financing of an underlying without having to sell it.

In its simplest form, a CLN is a promise of regular coupon payments and redemption of principal at maturity, just like an ordinary bond. However, the payments are linked to the performance of a reference credit security. For example, Bank A issues a CLN on the default risk of Company C with a nominal amount of €100 million. The note pays interest at 100 bp above a benchmark such as Euribor. If no credit event occurs on Company C, A repays the full nominal amount at maturity, and the investors incur no losses. Conversely, if a default event does occur before maturity, the note is called and coupon payments cease. With early redemption, the investor can receive either the reference securities once the default has been recorded or an amount equal to the market value of the reference underlier after default. In short, a CLN is a synthetic defaultable bond (see Exhibit B).

### Exhibit B - Credit Linked Note

Entity A issues a CLN on reference issuer C. Investor B buys the CLN as if it were buying a risky bond with embedded options on the default risk of Company C.

- **Premium** $x$ bp + Nominal
- **Nominal + coupons**
- **Asset after default**
Futures and options on credit spreads

Credit spread futures allow an investor to take a forward position on a yield differential. They are very similar to interest rate forwards. To protect himself from an issuer downgrade, a bondholder simply has to sell a future. If his bond depreciates during the life of the contract, the symmetrical payoff in the futures market offset the loss in the bond market. As a rule, these forward sales are automatically cancelled if the underlier defaults.

Credit spread options are used to take a position on a future credit spread relationship and give the investor a non-linear risk profile. They are very similar to options on equities or interest rates insofar as they entitle the investor to buy (call) or sell (put) a risky bond at a given spread (strike), either at or until a specified date (European-style or American-style, respectively). The option must be clearly defined: a call spread option that allows an investor to profit from the widening of a spread is in fact a put on a risky bond.

Note that a put option on a bond gives an investor enormous flexibility because it covers, in particular, the default risk of the reference issuer. When default occurs, the price of the issuer's bonds plummets, sending the put deeply in the money. For this reason, the premium of an at-the-money put will be higher than that of the call with the same strike and maturity; the difference includes the CDS premium.

Total return swap

With a total return swap (also known as a total rate of return swap), investors can synthetically replicate exposure to a reference asset that incorporates a credit risk. The swap transfers all the returns on a financial asset (interest, dividends, value changes) during a pre-defined period. Payments for value-changes occur at periodic interim dates - coupon payment dates - and also at maturity. In return, the protection seller makes regular coupon payments equivalent to a reference rate plus a spread (see Exhibit C).

The financial interpretation of a TRS is interesting. Entity A is exposed to Entity C's credit risk, and transfers it to Entity B. In practice, A pays B all the cashflows on the security issued by C, together with any net appreciation in value, and B pays A for the funding of the investment on C (Euribor plus a margin) together with any net depreciation in value. As a result, the TRS allows Entity A to single out and eliminate C's default risk.
- Total return linked notes

A total return linked note is a kind of synthetic bond combined with a TRS: like a CLN, it has to be funded at the initial date. Issues of total return linked notes synthetically replicate the economic performance of an underlying asset. Take a concrete example. A bank issues a $600 million synthetic bond on a major retailer, and some of its clients sell their exposure on that company. The assets are then pooled in a special-purpose vehicle (SPV) that is set up to structure the product (see Part Two for more information). The bank enters into a TRS with the SPV whereby the vehicle pays the asset’s economic performance to the bank, which in return pays an annual coupon 65 bp over the 10-yr T-bond yield. At the same time, the SPV issues a note linked to the retailer’s default risk over 10 years with a $600 million principal and a coupon equal to the SPV’s payoff, i.e. a 65 bp spread. Basically, the $600 million transited via the SPV and was paid to the bank’s clients, who thus sold off their credit risk on the retailer (see Exhibit D).
The success of this package is due to the generous return on the note, namely a 65 bp spread compared with an average 45 bp on the retailer’s debt. This excess return is possible for two reasons. First, the securities pooled in the SPV generate a higher return than the retailer’s bond debt; second, buying a structured synthetic security over the counter would entail a higher liquidity risk compared with a plain-vanilla bond.

This type of product exists mainly because it has a higher spread than conventional securities. Without this, investors would prefer to buy the underlier directly.

- **First to Default**

In contrast to the credit derivatives examined so far, the underlier of a “first to default” protection is a group – or basket – of risky securities.

With a “first to default”, credit risk can be transferred synthetically by means of a swap. A counterparty that wishes to get rid of credit risk buys protection from a seller in return for a periodic premium. The seller undertakes to compensate the buyer for any loss on the first security in the reference basket to experience a credit event. A bank can use a “first to default” to transfer some of the risk on its loan portfolio. And in certain cases, by purchasing this type of protection it can reduce the amount of regulatory capital it needs to hold for the Cooke ratio. For investors, a “first to default” makes it possible to take a synthetic credit position on a basket of entities without having to fund it upfront. Compared with the CDSs of each entity in the basket, the “first to default” provides leverage because of the overall number of entities present. Companies can use a “first to default” to manage their exposure to specific customers and suppliers, or to hedge their country risk.
Other contracts containing credit derivatives

A growing number of contracts now contain option clauses on credit. These are products with yields linked to an issuer's rating or credit spread – notably bonds with coupon step-ups, which are pegged to the issuer's rating. This type of issue has become very popular since 2000, particularly in the telecoms sector, because it reassures investors in the event of downrating. Take the example of two France Télécom issues: the 5.75% due 2004 and the 6.75% due 2008. Both contained clauses linked to France Télécom's ratings from Standard & Poor's and Moody's. The coupons rise (fall) 25bp per notch and per agency if the rating is downgraded (upgraded).

1.3 Legal and operational aspects of credit derivatives

Despite an all-out explosion in credit derivative issuance in recent years, further development has been hindered by the unwieldy legal, operational and regulatory aspects of this activity.

Legal status

In this paragraph we describe how French law applies to pure credit securities pursuant to Article 3 of the Financial Activity Modernisation Act of 2 July 1996. The three main types of products covered by the legislation are credit default swaps, spread options and total return swaps.

A derivative instrument on a credit event is based on a default mechanism which, when activated, entitles the protection buyer to a payment under precise and predefined conditions. Among the events that can trigger payment are the downrating of the debtor, non-performance of a contractual obligation, failure to comply with specific financial ratios, and, naturally, non-payment or bankruptcy. Credit spread derivatives are transactions intended to protect investors from fluctuating credit margins. They are similar to options or interest rate swaps. A total return swap allows one party to fully transfer changes in value of a reference underlier to another party.

For the above contracts, the 1996 Act clearly concludes that credit derivatives come under the category of "financial futures", the definition of which is entirely separate from the type of underlying asset. More precisely, credit derivatives are basically treated in the same way as swap contracts. This is corroborated by the International Swaps and Derivatives Association (see below).

However, it has often been wondered if other legal classifications of credit derivatives could be suggested by market practitioners rather than lawyers. This calls for a precise situation report, with reference to the prevailing legislation. One

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5 See Kolifrath (1999).
key question is whether buying protection through a credit derivative can be considered as an insurance transaction from a legal standpoint.

An insurance policy is a contract through which an insurer undertakes to compensate another person (the insured) for personal injury or damage to property, in return for a consideration (the premium). From a technical perspective, an insurance transaction is based on the principle of risk pooling. From the point of view of the insured, only actual injury or damage can be compensated, and he or she must do everything to prevent the occurrence of the insured risk. Therefore, there are two key differences between a credit derivative and an insurance transaction. First, credit derivatives infringe the principle whereby policyholders share the risk burden and contribute jointly, in the form of premiums, to indemnifying those experiencing injury or damage. Second, a credit derivative can be seen as both a hedging transaction and a speculative transaction. This contravenes the indemnity principle, which states that an insured cannot make a profit because he or she is entitled to compensation solely for actual injury or damage. For these reasons, credit derivatives cannot be treated in the same way as insurance contracts.

Legal considerations form the first barrier to the use of credit derivatives. The situation is compounded by the operational constraints that arise when a credit derivative contract is signed.

**Operational aspects**

Before entering into a credit derivative contract, two precautions need to be taken.

- Both the buyer and seller of a credit derivative must make sure they do not breach a confidentiality requirement when the credit event concerns a private event, such as failure to meet a loan repayment. Divulging this type of private event to a credit derivative counterparty with the intention of obtaining a payment can constitute an outright breach of the confidentiality requirement.

- Buyer and seller must also take care not to break the law on insider trading. An entity that receives privileged information might be tempted to purchase protection against the default risk of the reference company. Furthermore, the disclosure of this information could give rise to a credit event.

In 1999, the International Swaps and Derivatives Association (ISDA) published its Short Form Confirmation for CDS transactions, based on existing confirmations for derivatives. The document gives details of the product's maturity, the reference asset, the two parties to the contract and the characteristics of the swap's fixed and floating legs. It entails the following actions:

1) Determine the contract's key dates (e.g. maturity), as well as the counterparties, the reference security and its issuer, and business day conventions;
2) Describe the fixed rate payer;
3) Define the obligations of the floating rate payer, in particular the situations in which payment is triggered;
4) Provide for settlement (cash or physical delivery of securities);
5) Give details of specific clauses, e.g. changing the underlier if it becomes insufficiently liquid.

Each of these points is accompanied by a set of definitions that are incorporated into all credit derivative contracts. The publication of the ISDA Short Form Confirmation and definitions has made a significant contribution to standardising and enhancing the legal security of the default swap market. But has not completely removed the risks related to these products insofar as the confirmation relates to default swaps only. Also, legal risk seems to be an ever-present threat to credit derivative contracts as a whole: in January 2001 the Basel Committee recommended a 15% capital allocation in addition to the Cooke ratio to take account of that risk.

**PART TWO**

**STRUCTURED CREDIT DERIVATIVES**

11.1 Definition

Structured credit derivatives have developed in parallel to derivatives on a single underlier. Whereas the products presented in Part One are used to manage securities on an individual basis (with the exception of "first to default" protection), the products discussed here approach credit from the angle of a risk portfolio. Structured credit derivatives can replace an entire loan portfolio because they contain the funding and diversification components that typify such portfolios. Let us analyse the behaviour of securities backed by a portfolio of credit bonds (i.e. of securities). When all potential default scenarios for the underlying securities are taken into consideration, the number of possible cashflows from the portfolio is enormous and must be determined by a general rule. Moreover, some securities might be shorter dated than the derivative. In this case, when the security matures the flows can be reinvested in other underlyings on the basis of criteria set when the credit derivative is issued. In general, these derivatives have their own legal personality which is totally separate from that of the originating bank. They do not come within the framework of over-the-counter transactions that characterises the swaps presented in Part One. We will review three products, namely collateralised bond obligations, collateralised loan obligations and synthetic securities. They all have two features in common: they are fully funded (the investor pays the nominal amount at the initial date, unlike a CDS, for example, where the...
nominal flow is paid only in the event of default) and they take the form of bond tranches with a credit rating.

II.2 Collateralised bond obligation (CBO)

A CBO is a fully funded product that operates on the basis of prioritised interest and principal payments.

CBOs are structured products guaranteed by a pool of bonds and related to securitisations. They are issued by an SPV, that is to say a company set up with a single aim. SPVs are generally located in countries like Jersey, Ireland, the Netherlands and Luxembourg, where financial vehicles are lightly taxed and/or administrative formalities are streamlined but where the legal framework is robust. One of the motives in setting up an SPV is to make it legally independent from the originating bank and to issue notes that have a higher rating than the bank's. The supporting company, whose name the CBO bears, issues different tranches that will be sold in the market to investors. The tranches – usually between three and five – that comprise the SPV's liabilities are prioritised and offer different yields. The bank that arranges the CBO chooses one or more agencies to rate each of the tranches. From a purely financial viewpoint, the tranches resemble bonds: they pay regular coupon interest over several years and are redeemed at their nominal value. When the CBO is launched, the proceeds from the sale of the tranches are used to acquire the assets of the SPV. These consist of a pool of bonds purchased by the vehicle in the secondary market. One or two asset classes are usually grouped together within a CBO. These may be investment grade bonds (rated between Aaa and Baa3) or high yield bonds (Ba1 and C), issued by governments or corporates and denominated in one or more currencies. The asset portfolio is selected and administered by an experienced asset manager charged with optimising the yield using the investment rules defined during the structuring process. Exhibit E: gives an example of how a CBO is organised.
Exhibit E – Redwood CBO launched by HSBC on 28/11/2001

The diagram shows the operating structure of Redwood, a €240 million CBO arranged by HSBC-CCF. The collateral, managed by Franklin Templeton Investment, consists of US and European high-yield bonds as well as emerging sovereigns. Some 55% is denominated in USD and 45% in EUR. Cash flows are converted to euro via a currency swap.

The coupons from Redwood’s asset portfolio are used to honour the CBO’s liabilities by paying interest on each euro-denominated tranche. The senior tranche, rated Aaa by Moody’s and AAA by Fitch, accounts for 70.25% of the issue and pays 6-mo Euribor plus 55 bp; the second senior tranche, rated A2, pays a 6.37% coupon and accounts for 14.65% of the issue; and the mezzanine tranche, rated Ba1, makes up 4.85% of the issue and pays a 11.18% coupon. The subordinated tranche receives all the excess cashflow, i.e. the coupon payments over and above those due to the senior and mezzanine tranches. The coupons are paid six monthly until the CBO matures on 15 December 2011.

The collateral assets can be summarised in two key characteristics: (using Moody’s methodology).

- the average rating, which expresses the default probability of the bonds in the portfolio;

- the diversity score, reflecting the number of independent securities in the portfolio.

These criteria are essential for rating the senior and mezzanine tranches once the respective proportions have been determined.
The main reason for this type of arrangement is to reconfigure the return/risk profile. All the tranches issued by the CBO have the same average return and risk as the initial portfolio. However, each tranche has its own rate of return and risk. Because of the "portfolio" effect - i.e. diversification - it is possible to issue senior and mezzanine tranches that each yield more than identically rated bonds traded in the market. For example, the Aaa rated tranches have an excess yield of several tens of basis points. This type of structure is known as an "arbitrage CBO".

The subordinated tranche receives all the excess cashflows, that is to say the cashflows paid by the assets minus the coupons owed to the senior and mezzanine tranches. The subordinated tranche has very high expected returns (around 20% p.a.), which cannot be matched by any ordinary debt obligation in the market. It allows investors to leverage the portfolio's yield (and risk).

When an investor buys a single bond, he has only two possibilities as far as a default event is concerned. Either there is no default, in which case he receives the payoff expected at the time of purchase (e.g. Euribor +250 bp for a high yield bonds); or a default occurs and he incurs a loss that depends on the bond's recovery rate. If the recovery rate is 35%, then the loss is 65% of the investment outlay. This means that the payoff is not wholly "guaranteed" at the time of the investment but depends on the issuer's probability of default. By contrast, if the investor invests in a credit portfolio, the default situation is very different: the loss engendered by a default is proportional to the issuer's percentage share of the portfolio. If there are 50 bonds, a default on one of them (with a 35% recovery rate) creates a loss of 65%/50=1.3% of the collateral. If the bonds originate from different industrial sectors and different geographical areas, default events are highly decorrelated. This limits the probability of a chain reaction and sharply reduces the risk of extreme loss.

With this in mind, Moody's developed a methodology for calculating the probability of losing a fraction of a portfolio because of a default chain reaction. The portfolio is represented by a portfolio of D independent securities with the same rating (the portfolio's average rating) and maturity (its average maturity). Moody's examines how each of the possible scenarios would impact on the securities. The method can be used to compute the average loss that could affect each of the notes issued by the CBO. The average loss is converted to a rating, which informs investors about the expected risk. This binomial method is outlined in Exhibit F.

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Exhibit F - Binomial method applied to a CBO

Assuming the asset portfolio contains D independent securities, all default events ranging from 0 to D can be envisaged. One scenario consists in calculating the cashflows received by the senior, mezzanine and subordinated tranches for a given number of defaulted securities. Take the example of a portfolio comprising 20 independent securities (D=20) with a 10-yr default probability of 10%, denoted by p and derived from the portfolio’s average rating.

<table>
<thead>
<tr>
<th>Diversity score</th>
<th>Assets</th>
<th>Liabilities</th>
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</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-yr default</td>
<td>10.0%</td>
<td>Senior tranche 80%</td>
</tr>
<tr>
<td>Recovery rate</td>
<td>50.0%</td>
<td>Mezzanine tranche 15%</td>
</tr>
<tr>
<td>Maturity</td>
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<td>Equity tranche 5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Default scenarios</th>
<th>Probability</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. defaults</td>
<td></td>
<td>Senior</td>
</tr>
<tr>
<td>0</td>
<td>(1-p)²⁰</td>
<td>0.0%</td>
</tr>
<tr>
<td>1</td>
<td>20 p (1-p)¹⁹</td>
<td>0.0%</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>C²⁰₀ p¹⁰ (1-p)¹⁰</td>
<td>3.8%</td>
</tr>
<tr>
<td>...</td>
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<td>19</td>
<td>20 p¹⁹ (1-p)</td>
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<td>27.9%</td>
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<tr>
<td>Average loss</td>
<td>0.00013%</td>
<td>4.3%</td>
</tr>
</tbody>
</table>

The "10 out of 20 default" scenario occurs with a probability of 0.0006%, calculated with the binomial formula.

Given the proportions and coupon rates of the senior and mezzanine tranches, as well as the recovery rate of the defaulted securities, we compute losses of 3.8% for the senior tranche and 63.4% for the mezzanine tranche under this scenario.

The rating is obtained by computing the average of the losses under all scenarios. Here we obtain loss levels of 0.00013% and 4.3%. We compare these with the historical default tables in order to infer the rating. As a general rule, the subordinated tranche that receives the excess coupons is not rated.
Exhibit G gives an example of CBO whose assets consist of two correlated securities and that issues two tranches.

**Exhibit G - Simple example of a CBO**

Consider a CBO that issues two tranches. The senior tranche A receives the cashflows first; and the B tranche receives all the excess cashflows. Assume that the CBO’s assets comprise two securities with the same par value and with the same default probability of 9%. Four scenarios may occur:

- neither security defaults
- only the first security defaults
- only the second security defaults
- both securities default

Assuming the two securities have a face value of €50 and a 30% recovery rate, they yield €15 in the event of default.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Tr. A</th>
<th>Tr. B</th>
</tr>
</thead>
<tbody>
<tr>
<td>ND ND</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>ND D</td>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>D ND</td>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>D D</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>

*Expected loss*  
0.32%  12.28%  1.14%  11.46%  1.96%  10.64%  2.78%  9.82%

*Rating*  
A1  B2  Baa1  B1  Baa1  B1  Baa3  Ba3

The above table shows the probability of occurrence as a function of $\rho$, the default correlation, and the average loss per tranche for all four scenarios. Taking the average loss and using Moody’s tables, we can work out a rating for each tranche. When the assets are weakly correlated, the senior tranche is well rated (A1) and the subordinated tranche is poorly rated (B2). When, by contrast, the default correlation is high, the senior tranche’s rating deteriorates (Baa3) while the subordinated tranche’s rating improves (Ba3).

Therefore, to issue a well-rated senior tranche, the CBO’s assets should be composed of securities that are as weakly correlated as possible.

II.3 Collateralised loan obligation (CLO)

CLOs are very similar to CBOs, but the collateral consists of a portfolio of bank loans rather than a pool of bonds. The methodology is the same; all that differs is the motive for issuance.

Whereas CBOs are designed for clients seeking a yield pickup relative to ordinary securities, CLOs are issued by banks that have decided to sell on some of the loans they have made to their clients. In this way, they can maintain their commercial relationship with those clients because they can reduce their credit
risk on them. With so-called balance sheet CLOs, a bank can reduce its capital requirement by re-selling loans and, subsequently, can increase its return on regulatory capital. Under the rules of the Bank for International Settlements (BIS), capital must be allocated for each commitment (the actual amount is determined by the Cooke ratio). For banks the appeal of CLOs is not so much economic – they can accept to issue tranches offering a higher average return than that on their loan portfolio – as strategic: a CLO programme increases the return on regulatory capital while allowing a bank to develop a commercial relationship with former clients. This can warrant paying a return on a CLO that exceeds the return on the underlying loans. The economic cost is offset by a higher return on capital, achieved through a sharp reduction in the capital allocation.

**Exhibit H** shows the capital saving made by a bank via a CLO issue. Based on the average outcome of recent issues, the capital consumption of a loan portfolio is divided by four.
Where a portfolio of loans sold on through a CLO has a high average rating, a bank can issue a substantial proportion of well-rated CLO notes and retain only a small portion of the subordinated tranche (usually equivalent to 2% of the issue's nominal amount).

The senior and mezzanine tranches are sold to investors. Regardless of how the portfolio performs, the maximum loss experienced by the issuing bank, which holds only the subordinated tranche, will not exceed the amount of that tranche. These subordinated notes, which assume first-loss risk, require a capital allocation equivalent to the size of the tranche (2%). If the bank had not arranged a CLO, its capital allocation for a portfolio of corporate loans would have been 8% (BIS rules, 1988).

The table below shows how a portfolio of loans originally carried on a bank's balance sheet is sold on through a CLO, with the issuing bank retaining only the subordinated tranche.

<table>
<thead>
<tr>
<th>Nominal</th>
<th>Initial portfolio</th>
<th>Senior tranche</th>
<th>Mezzanine tranche</th>
<th>Subordinated tranche</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000,000</td>
<td></td>
<td>80,000,000</td>
<td>18,000,000</td>
<td>2,000,000</td>
</tr>
</tbody>
</table>

Since the capital allocation is smaller (2,000,000 instead of 8,000,000; the average for recent transactions shows a four-fold reduction in capital consumption for a loan portfolio), the overall return on the bank's capital improves despite the cost of issuing the CLO.

II.4 Synthetic securitisation

Synthetic securitisation is the most highly evolved form of credit derivative and applies mainly to bank loans. In a classic securitisation deal – a CLO – and, increasingly, in arbitrage CBOs, the on-sale of a loan or a claim can raise legal problems as regards the validity of the sale. The purpose of synthetic securitisation is not to make a sale but to construct a product derived from a pool of claims. The derivative, arranged between the originating bank and a financial vehicle, stipulates that the vehicle will reimburse the bank for loan loss in excess of a certain threshold (the derivative is not activated unless the loss exceed 2% of the nominal value of the loan portfolio).

In practice, an SPV issues prioritised notes on the market (cf Exhibit I). The cash proceeds of this issue are used to acquire a collateral portfolio for the default-swap (either Aaa-rated securities or liquid bank-issued securities). With this portfolio of liquid and riskless securities, the SPV engages in a credit derivative arrangement with the originator whereby it undertakes to make good the loss if
a claim due to the originator goes into default. To honour its guarantee, the SPV sells part of the collateral portfolio.

One interesting feature is that the notes issued under this arrangement usually account for no more than 10% of total outstanding. For example, the subordinated tranche can cover losses of up to 2% of the nominal value; the mezzanine tranche covers between 2% and 5%; and the Aaa-rated senior tranche covers losses of more than 5% but less than 10%. For the remaining 90%, the originating bank enters into a default swap with other banks, which guarantee the "catastrophe" risk – dubbed "Super Aaa" because the probability that the counterparty to the default swap will be required to cover losses is lower than the default probability of a Aaa-rated tranche. The default swap does not kick in until the senior tranche has covered its portion of the defaults and used up its entire nominal value. Since this tranche is Aaa-rated, the risk that the default swap will be activated is minute.

At maturity, the investors are repaid by liquidating the collateral portfolio's assets. Prior to this, any losses paid out by the SPV to the originator through the credit derivative are deducted.

By entering into the derivative, the issuing bank undertakes to pay the SPV a default premium on a periodic basis (three-monthly or six-monthly). The return on the tranches consists of the coupon interest from the securities in the collateral portfolio and the default premium paid by the originating bank. Throughout the life of the structure, say 7 years, the tranches will earn a periodic coupon, with the face value being either redeemed at maturity or amortised progressively. If a high proportion of the loans protected by the derivative go into default, the low-priority tranches will have a smaller than expected return.
This principally applies to the subordinated tranche, then to the mezzanine tranche and exceptionally (i.e. if the default rate is much higher than expected) to the senior tranche.

The rating of the tranches is determined in more or less the same way as for a conventional CBO, by analysing the risk diversity of the CDS underliers and their average default probability.

One of the major advantages of synthetic securitisation is the legal simplicity of the structure, stemming from the fact that there is no need to sell the underlying claims. These can take a variety of forms, including bank loans (the majority of cases), long-dated bonds, receivable-related claims and consumer credit. The cost of setting up a synthetic securitisation is therefore lower than for a conventional securitisation deal, where the problems of transferability have to be analysed (notably the question of transferring value-added tax).

In addition, the return on the super senior tranche of the default swap costs the SPV less than the payout on the tranches fully funded by investors. This is because the funds are used to acquire collateral which usually returns less than Euribor, and the difference is made up by the SPV. This excess cost does not show up in the remuneration of the CDS insofar as the counterparty to the super senior tranche is not called on unless the underliers are in default.

II.5 Credit derivatives and capital allocation

One of the reasons for the development of credit derivatives is their use in the sphere of regulatory capital allocation. Regulations vary from one country to another and are set by national banking regulators. (In France, they are updated annually.) The rules governing the recording of credit derivatives in the trading book are fairly stringent. For this reason, we shall deal only with the general rules, i.e. those governing credit derivatives in the banking portfolio (investment portfolio). In this case, the institution that guarantees a risk must allocate a quantity of capital equivalent to that risk.

For example a seller of default protection on the Alcatel 12 October 2004 bond (the risk "buyer") is exposed in the same way as if it held the bond directly. It must therefore allocate a quantity of capital equivalent to 8% of the underlier's face value to the derivative. However, a bank that holds a security and actually buys default protection for it (the risk "seller") will be exposed to the default of its swap counterparty and will therefore allocate a quantity of capital that depends on its CDS counterparty (generally just 1.6% if the counterparty is a bank). Exhibit J gives an example of how a CDS can be more profitable than buying a security directly.

---

Consider corporate issuer X offering a return of 70 bp over Euribor. Two banks contemplate buying X’s bond:

- Bank Y, which funds at Euribor -10 bp.
- Bank Z, which funds at Euribor +20 bp.

The following table shows the return on equity for both:

<table>
<thead>
<tr>
<th>Bank</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on the Asset</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Funding</td>
<td>10</td>
<td>-20</td>
</tr>
<tr>
<td>Total return</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td>Capital allocation</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>ROE</td>
<td>10%</td>
<td>6.25%</td>
</tr>
</tbody>
</table>

ROE is calculated as the total return divided by the capital allocation, expressed as a percentage. For example, for an outlay of €100, Bank Y’s ROE is:

$$\frac{100 \times 80}{100 \times 8} = 0.8/8 = 10\%$$

The two banks consider an alternative solution: Bank Y buys X’s bonds and does a credit default swap with Bank Z to protect itself against default risk, for a premium of 60 bp.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on the Asset</td>
<td>70</td>
<td>-60</td>
</tr>
<tr>
<td>Funding</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Return on CDS</td>
<td>-60</td>
<td>60</td>
</tr>
<tr>
<td>Total returns</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>Capital allocation</td>
<td>1.6%</td>
<td>8%</td>
</tr>
<tr>
<td>ROE</td>
<td>13%</td>
<td>7.50%</td>
</tr>
</tbody>
</table>

In this way, even though Bank Z earns less for the X risk than it would if it bought the bond directly (60bp versus 70bp), Bank Y and Bank Z both have a higher ROE than they would with direct purchase.

Recently commercial banks have issued lots of CLOs– the number of issues rose more than 50% between 1999 and 2000 – for two main reasons. They free up liquidity on overexposed clients and they reduce capital allocations. The regulatory allocation generally falls from 8% of the nominal amount (for corporate loans) to around 2% because it is limited to the subordinated tranche of the CLO.

Under new BIS regulations, due to come into force in 2005, regulatory capital allocations will depend on the rating of the tranches. The following table shows the old and new allocations.
Note the increasing emphasis placed on the credit rating. This will determine the capital allocation, which will depend on the risk actually incurred.

PART THREE

LATEST DEVELOPMENTS IN QUANTITATIVE MODELLING

The aim of this last section is to present the latest developments in the quantitative models used to evaluate credit derivatives. As a preamble, we recall the different types of credit model. We then go on to show how risk-neutral pricing theory applies to default swaps. We conclude with a detailed description of a method that provides a confidence interval for spread option prices.

III.1 Two approaches to credit risk modelling

Claire Gauthier (1998) has produced a comprehensive review of credit models, which we review briefly below.

There are two main quantitative approaches to modelling credit risk. The first dates from the work of Merton (1974), who considered that a defaultable asset is contingent on the value of the firm and can therefore be priced using option theory. In this type of model, the firm's value follows a random process in continuous time, and default occurs when the process hits a pre-defined boundary. These models are suitable for situations involving several bonds issued by the same company, as is the case with convertible bonds, for example. However, since the firm's value follows a continuous process, default is foreseeable because it becomes increasingly certain as the value nears the boundary. This results in an unrealistic term structure of spreads that tends to zero for short maturities.
The second main approach involves jump models. Here, default occurs at a random date which is the date of the first jump in a Poisson process. Unlike the structural approach, default is not foreseeable and the analytical framework is therefore more realistic. That said, the structural interpretation of default is lost. For example, the joint description of the way in which different assets of the same issuer evolve is bound to be exogenous. One further refinement of these intensity models, initiated by Jarrow, Lando and Turnbull (1997), consists in introducing different levels of rating, where the dynamics of the company's rating are governed by transition matrices (see below).

Exhibit L - Risk-Neutral Default Probability

The intensity-based approach is widely used because the parameters can be easily derived from market prices. Consider for example a risky zero-coupon bond with maturity $T$ and riskless interest rate $r$. If $s$ is the bond's spread over riskless rate and $\omega$ is the recovery rate in the event of default, then the risk-neutral default probability $q$ satisfies the no-arbitrage condition:

$$e^{-sT} = 1 - q + q\omega$$

This relation shows that the spread required for a risky investment must offset the average loss engendered by the credit risk. Accordingly, the risk-neutral probability of default is a function of two variables: the ratio of the risky and non-risky discount rates, and the recovery rate. Using market prices, we can then work out the implied default probabilities in order to price risky securities that are more complex.

III.2 Pricing a credit default swap

The price of a CDS can be obtained by explicitly constructing a hedge from instruments available in the market. Assume that two counterparties enter into a CDS with a two-year maturity. The contract states that the reference asset is a risky bond issued by a third party. The protection buyer makes regular payments as long as the issuer of the reference bond does not default. If the bond defaults within the two-year period, the protection buyer delivers it to the protection seller in exchange for its par value plus interest. In this example, there is hardly any difference in terms of risk between the CDS and the bond. To price the CDS, we determine the size of the fixed payments made by the protection buyer to the protection seller. In this case, a credit model is not needed; we simply construct a static hedge on the CDS by purchasing the reference assets outright at the initial date and holding them until the swap matures.

This hedge is not the same for the protection buyer as for the protection seller. The buyer must be long the bond if it raises funds in the market at Euribor and buys the risky bond, which yields Euribor+$X$. This hedge would earn the protection buyer a net margin of $X$. If the reference bond does not default, the buyer simply settles its position at maturity; otherwise, it delivers the hedge when default occurs.

---

A similar rationale applies to the protection seller, who has to short the bond. It borrows the bonds in the repo market and sells them on the open market. Assume that in order to borrow the bonds, it lends the nominal value of the bond at Euribor -Y. There are two reasons why the value of Y could prove to be very high. First, the loan is backed by a bond, and the borrower therefore expects a low borrowing rate. Second, since the repo market in risk bonds is inefficient, Y could be between 20 bp and 150 bp. In this case, the protection seller sells the risky bonds and pays the securities lender Euribor+X. The net cost of the hedge is therefore a spread of X+Y.

So we can see that the hedging cost is not symmetrical, depending on the point of view. We simply obtain a bid/offer range for the CDS. The final price will depend on the counterparties and their aims. As a general rule, pricing a CDS is not so simple; for example, the bond may have a longer maturity than the CDS, or financing rates may vary over time.

Spreads on credit swaps differ from bond spreads for other reasons, which can be split into two categories. First, there are fundamental factors stemming from inherent differences between bonds and default swaps - for example, how the products are treated on the balance sheet; whether the CDS contract contains counterparty risk; or whether the bonds has a step-up coupon pegged to the issue's rating. Second, market-related factors such as the cost of trading in the repo market, liquidity, and new issuance also play a part. What is more, the credit events that trigger payments on a CDS are broader in scope than the default events recognised by rating agencies, meaning that a CDS can be activated even if no default occurs. This can happen, for example, if constraints arising from financial ratios are not satisfied at a given moment, even though the bond interest and principal has been paid.

III.3 Uncertain parameter models

The above paragraph explained why it is pointless to try and assign a single price to a credit derivative. In this paragraph we will look at spread options; we assume that the underlying securities (risky bonds) are liquid and we discount transaction costs. In this setting, the size of the bid/offer range for these options is attributable to the incompleteness of the market: with only a single risky bond available, it is impossible to hedge both the fluctuations of the spread and the risk of rating transition.

The following model is described in meticulous detail by Brunel (2001). Assume the market to be composed of one risky bond and one riskless asset. For the sake of simplicity, the riskless interest rates are assumed to be constant and the term structure flat. The price of the risky bond and the spread are linked by the relation shown in Exhibit M. Accordingly, the risky bond's price fluctuations can be construed as spread fluctuations.

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9 See Tolk (2001).
Exhibit M – Spread Fluctuations

As described in Exhibit A, there is a simple relationship between the price of a risky zero-coupon bond $B(t,T)$ with maturity $T$ at date $t$ and the spread $s$. If the riskless rate is denoted by $r$:

$$B(t,T) = e^{-(r+s)(T-t)}$$

Therefore, the bond price fluctuations are reinterpreted as spread fluctuations. Assuming that the spread variations follow a normal process in continuous time, Ito's lemma lets us express the risky bond's price dynamic as a function of the spread's dynamic. If at date $t$ the issue is rated $R_t$, and the standard deviation of spread variations is a function $\sigma(R_t)$ of the rating, then the dynamic of the risky zero-coupon bond under risk-neutral probability is:

$$\frac{dB_t}{B_t} = r dt + \sigma(R_t)(T-t)dW_t$$

Although the dynamics of the spread can be easily modeled, the dynamics of the rating are harder to grasp. Rating agencies such as Moody's and Standard & Poor's supply historical probabilities for rating transition and default for a one-year timeframe. These probabilities are obtained by observing a broad sample of rated companies and comparing their ratings year-on-year (see Exhibit N).

Exhibit N – Transition Matrices

The matrix below shows the probability of transitioning from one rating to another during a one-year period. A company originally rated A has a 0.74% probability of being rated BB one year later.

<table>
<thead>
<tr>
<th>Initial rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>90.81</td>
<td>8.33</td>
<td>0.68</td>
<td>0.06</td>
<td>0.12</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AA</td>
<td>0.7</td>
<td>90.65</td>
<td>7.79</td>
<td>0.64</td>
<td>0.06</td>
<td>0.14</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>A</td>
<td>0.09</td>
<td>2.27</td>
<td>91.05</td>
<td>5.52</td>
<td>0.74</td>
<td>0.26</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>BBB</td>
<td>0.02</td>
<td>0.33</td>
<td>5.95</td>
<td>86.93</td>
<td>5.3</td>
<td>1.17</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.14</td>
<td>0.67</td>
<td>7.73</td>
<td>80.53</td>
<td>8.84</td>
<td>1</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.11</td>
<td>0.24</td>
<td>0.43</td>
<td>4.68</td>
<td>83.46</td>
<td>4.07</td>
<td>5.2</td>
</tr>
<tr>
<td>C</td>
<td>0.22</td>
<td>0</td>
<td>0.22</td>
<td>1.3</td>
<td>2.38</td>
<td>11.24</td>
<td>64.86</td>
<td>19.79</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

By contrast, the issuer-specific probabilities of rating transition examined here are uncertain parameters that can, in principle, take any value between 0 and 1. If we now assume a spread option (e.g. a call on a risky bond), each parameter value will have a corresponding option price, and if we change the parameter values, the price will change. The point of an uncertain parameter model is to set a confidence interval for the parameters (in this case, the probability of transition) and use it to infer a confidence interval for spread option prices.

Consider an option on a risky bond. Its price depends on three variables: the date, the spread and the rating. This is because, assuming a constant spread, the option price depends on the volatility of the spread, which itself depends on the rating. We construct a hedging portfolio with a long position in one option and a short position...
in delta risky zero-coupon bonds. Then, by changing the values of the uncertain parameters in their confidence interval, we infer the best-case and worst-case movements in the portfolio depending on the value of the parameters. This leads to a partial derivative equation for the two bounds of the option price's confidence interval\(^{10}\).

Exhibit O - Price Equation

Assume an option on a risky zero-coupon bond with maturity \(T^*\) and payoff \(g(B(T^*,T))\). The price of this option at date \(t\) is denoted by \(C(t, B, R)\), where \(B\) is the price of the risky zero at date \(t\) and \(R\) is the issue's rating at the same date.

We have assumed that the uncertain parameters \(\lambda_{i,j}\) which determine the probabilities of transiting from rating \(i\) to rating \(j\) between \(t\) and \(t+dt\), lie between 0 and \(\lambda_{i,j}\).

The upper bound of the confidence interval of the price is a function \(C^+(t, B, R)\), which solves the equation:

\[
V_i + \frac{1}{2} v^2 (R) (T - t)^2 V_{BB} + r B V_B + \sum_{i=1}^{D} \lambda_{i,R} [V(t, B, R) - V(t, B, i)]^+ = r V
\]

where \(i\) is a whole number denoting the different values of the rating between 1 and \(D\), \(\lambda_{i,R}\) is the upper bound of the confidence interval of the uncertain parameters and \([A]^+ = \max(A,0)\). We see that since the rating is a discrete variable, this equation is actually a system of coupled partial derivative equations: there are as many levels of equations as there are levels of rating.

The equation system obtained in Exhibit O is solved numerically. For expositional ease, we have assumed a simple model with three levels of rating: R1 for investment grade, R2 for speculative grade and D for default. Our one-year transition matrix is described in Exhibit P.

Exhibit P - 1-yr Matrix used in the Three-Level Model

<table>
<thead>
<tr>
<th>Initial rating</th>
<th>Rating transition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R1</td>
</tr>
<tr>
<td>R1</td>
<td>99.7%</td>
</tr>
<tr>
<td>R2</td>
<td>3%</td>
</tr>
<tr>
<td>D</td>
<td>0%</td>
</tr>
</tbody>
</table>

By solving the equation system numerically, we obtain the price spread that gives a confidence interval for the option prices. We have considered an option on a bond rated R1. The first example concerns a call on a risky zero-coupon bond that entitles the option holder to buy the bond at a specified spread and date. The underlying zero has a 5-yr maturity and its spread can be negative in our model (this is realistic in the event that we consider the spread in relation to the swap rate).

---

\(^{10}\) See Brunel (2001).
For the numerical applications, we have considered the risky 5-yr zero-coupon bond. In the event of default, the recovery rate is 50%. The volatility parameter of the spread is 3% p.a. if the rating is R1 and 4% if R2. Each component of the generating matrix of default events for the risky bond is between 0 and 500 times the components of the generating matrix obtained from Exhibit P. In Exhibit Q, we calculate the bid/offer range for a call option on a defaultable bond.

Exhibit Q – Bid/Offer Range for a Call on a Risky Bond

Assume a call option on a risky bond (i.e., a put on a spread) with a 25 bp strike and 3-month maturity. This option entitles the holder to buy the underlying risky bond at a 25 bp spread in 3 months. The graph shows the option’s bid/offer range, expressed as a percentage of the bond’s nominal value, as a function of the spread at the calculation date. The dotted line represents the option payoff.

The second example relates to a spread option on the risky zero-coupon bond:
Consider the case of a spread call option on a risky bond with strikes of 25 bp and 0 bp and a 3-month maturity. This option is exactly equivalent to a call spread on an equity. The graph shows the option's bid/offer range, expressed as a percentage of the bond's nominal value, at the calculation date. The dotted line represents the option payoff.

Exhibit R - Bid/Offer Range for the Spread Option

The conclusion to be drawn from this numerical study of the credit derivative pricing model with uncertain parameters is that the bid/offer range ultimately obtained is acceptable. For the at-the-money spread option (Exhibit R), the size of the range is 25% of the worst-case price, which is totally comparable to the size of the bid/offer range in uncertain-volatility models and with market observations. In the case of a call on a risky bond, we obtain the same orders of magnitude.
CONCLUSION

Increasingly, conventional credit derivatives (presented in Part One) will be mass produced because of a higher level of standardisation. The work of the ISDA – in particular the 1999 publication of its definition of credit events that trigger the exercise of credit derivatives, as well as the Short Form Confirmation of a CDS – have made it possible to homogenise the creation of default swaps.

This five-page standard confirmation has contributed to the development of the CDS market, where underlying outstandings have reached $300 billion. As in recent years, these over-the-counter (OTC) products are likely to see annual growth of between 30% 50% in outstandings. This reflects growing concern among banks to manage their risk exposure which, according to future BIS regulations, will be the defining factor in terms of capital allocations and hence of return on equity.

Structured credit derivatives (Part Two) are likely to follow the same upward trend. CBOs allow investors to find products that are unobtainable elsewhere (such as junior tranches yielding 20% for a diversified and managed risk). They also make it possible to extract value from portfolio diversification by issuing tranches with different ratings, all of which outyield identically rated conventional bonds in the market.

CLOs have a direct impact on banks' balance sheets and are therefore set to develop significantly owing to growing pressure on credit institutions to improve their return on equity. If a bank is overexposed to a counterparty, it can start by using a CDS to eliminate this risk. When the amount outstanding in its loan portfolio becomes too large, the best solution is probably to issue a CLO that will allow it to pass the risk on to investors. CLO programmes generally have very large outstandings – several billion euro – and are more unwieldy to arrange than OTC derivatives because they have to be tailored to the needs of the bank and its investors. CLOs depend heavily on capital allocation regulations, so much so that the originator has the right to call some type of deal if the regulations change. The McDonough\textsuperscript{11} regulations make it possible to use internal models based on the same techniques for taking account of diversification and average default probability. This is likely to have a profound effect on the way banks view this matter.

\textsuperscript{11} See the report of the Bank for International Settlement (2001).
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