OPERATIONAL RISK

New results from analytical models

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Executive summary

- **Operational risk is the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events**
  - This definition includes legal risk, but excludes strategic and reputational risk
- **In the post-crisis environment, operational risks with unusual severities emerge regarding litigations**
  - Litigations with regulators
  - Litigations with clients
- **New risks emerge from the technological transition: cyber risk**
- **Regulators have recently published new guidelines and measurement standards for the capital charge measurement. OR capital charges are now often larger than market risk capital charges in large banks**
- **The Loss Distribution Approach (LDA) is the reference approach for measuring operational risk, but the range of practices is large and data are scarce**
  - Modelling choices (model risk) : severities, correlations, structure of the model
  - Calibration and validation issues
  - Few analytical results
- **Agenda**
  - Context: emerging risks and regulation
  - New results on OR correlations
  - New results from classification invariance

EMERGING RISK AND REGULATION

Operational risk is expensive

Rogue Trading

Barings (1995): $ 1.3 MM
Allied Irish Banks (2002): $ 691M
Société Générale (2008): € 4.9 MM
Caisses d'Epargne (2008): $ 938 M
Merrill Lynch (2009): $ 456 M
UBS (2011) : $ 2.3 MM
Credit Suisse (2012) : $ 2.85 MM

Reg. Rules Breach 2012-2014

OFAC
BNPP: $ 9 MM
HSBC: $ 1.9MM

Libor
UBS : $ 1.53 MM
Rabobank: $ 1.07 MM

Client litigations 2012-2014

Subprimes
BoA : $ 17 MM
JP Morgan : $ 13 MM

Payment Protection Insurance
Lloyds : $ 8.3MM
RBS : $ 2.67 MM
HSBC : $1.7MM
Barclays : $ 3.1MM

Terrorist Attacks

New-York (2001)
Madrid (2004)
London (2005)

Systems Failure

Knight capital (2012): $ 440M

Natural Disaster

Fukushima (2011)
Katrina (2005)
Sandy (2012)

Fraud

Madoff (2008)
« Madoff du var » (2011)

How do banks measure and manage operational risk?

- **Internal losses collection**
  - Most of the advanced banks have started to collect datas between 2000 and 2005
  - Useful for high frequency and low severity risk
- **External loss datas**
  - Several providers + one consortium gathering up to 70 large banks around the world (ORX)
  - External datas are not representative of the bank's risk => scaling issue
- **Scenario analysis**
  - Represent high severity low frequency risk or losses arising from multiple simultaneous events
- **Environment and internal control factors**
  - Quantification must embed the internal risk profile of the bank
  - Capture key risk factors in a forward-looking approach
- **OR management**
  - Key Risk Indicators (KRI)
  - Risk and Controls Self Assessment (RCSA)
  - Action and remediation plans
  - Insurance contracts

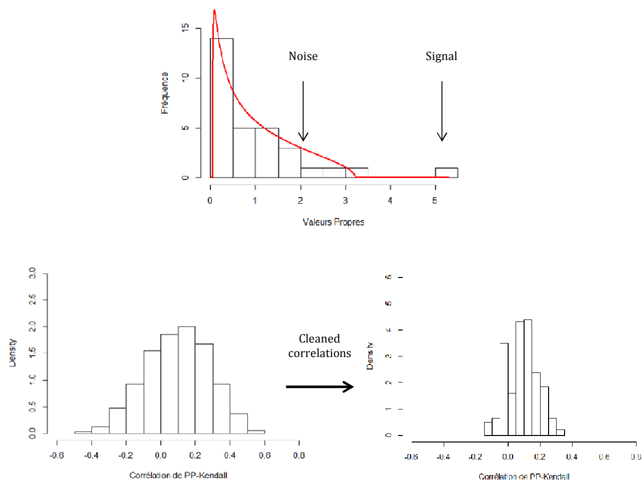
Requirements from regulation

- **The Basel regulation allows banks to use one of the 3 approaches**
  - Basic approach: capital charge proportional to the bank's gross income
  - Standard approach: capital charge proportional to the business lines' gross income
  - Advanced approach (AMA): Loss distribution Approach (LDA) or Scenario Based Approach (SBA)
- **In the AMA approach, the capital charge is equal to the 99.9% loss over 1 year**
- **Measurement of the capital charge must include the use of internal / external datas, scenario analysis and Environment and internal control factors**
- **EBA has issued guidelines regarding AMA frameworks**
  - The AMA perimeter should include OR linked to credit risk
  - Internal models will be constrained by the regulation
- **BCBS publications**
  - Consultative paper about the revision to the simpler approaches (basic and standard)
  - Review of the AMA framework expected in 2015

NEW RESULTS ON THE CORRELATION PROBLEM

Sound correlations vs. noise

Study based on ORX datas



Cell risk modeling

- **Aggregate losses computed from the OpRisk SAS Database are compliant with lognormal tails**
  - For a lognormal distribution, the parameters are linked to measurable quantities
  - The implied parameters are in a stable range of values for all confidence levels

Confidence level	Average	StDev
95%	98%	41%
97,5%	99%	39%
99%	107%	44%
99,5%	112%	46%
99,9%	124%	48%
All	107%	42%

$$\frac{\text{Expected value }(i)}{VaR_q\,(i)} = e^{\frac{\sigma_i^2}{2}+\sigma_i F_q}$$

- **Cell loss correlations are proportional to the number of events correlation (Frachot *et al.*, 2004).The correlation upper bounds depend on cells frequencies**

$$\mathrm{corr}(L_1,L_2) = \mathrm{corr}(N_1,N_2).\,e^{-\frac{1}{2}\sigma_1^2-\frac{1}{2}\sigma_2^2}$$

$$\mathrm{corr}(N_1,N_2) \le R = \sqrt{\frac{\min(\lambda_1,\lambda_2)}{\max(\lambda_1,\lambda_2)}}$$

- **Loss correlation upper-bounds from OpRisk SAS Database**
  - Average = 1.33%
  - Standard deviation = 1.61%
  - Maximum = 11.27%

- **The copula parameters are much lower than 10% on average**

Analytical model: assumptions and definitions

ASSUMPTIONS

- **Cell losses are lognormal**
- **One factor model**
- **Gaussian copula: pair-wise correlations may be different to each other**
- **We assume that the parameters are not dependent on the number of cells; the number of cells goes to infinity**

DEFINITIONS

- **Cell loss** $L_i = e^{\mu_i-\sigma_i\left(\beta_i F+\sqrt{1-\beta_i^2}\varepsilon_i\right)}$
- **Correlation** $\mu_{ij} - \beta_i.\beta_j$
- **Bank's loss** $L(F) = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} L_i = E\left[e^{M-\Sigma\left(\beta F+\sqrt{1-\beta^2}\varepsilon_i\right)}|F\right] = E\left[e^{-\Sigma\beta F+\Sigma^2(1-\beta^2)/2}|F\right].E\left[e^{M}\right]$
- **Bank's capital charge** $N.L(F_q)$ $F_q = N^{-1}(0.1\%)$
- **Stand-alone cell capital charge** $KSA_i = e^{-\sigma_i F_q}$

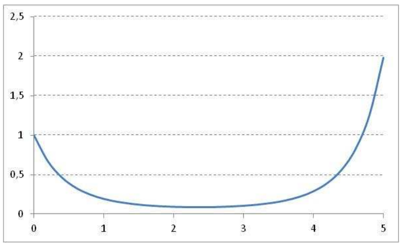
Homogeneous portfolio

- **The bank's loss is still lognormal**

$$L(F) = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} L_i = e^{-\sigma\sqrt{\rho}F+\sigma^2(1-\rho)/2}$$

- **Negative diversification appears when individual cell risk is larger than a given threshold**

$$\frac{N.L(F_q)}{\sum_{i=1}^{N} KSA_i} \xrightarrow{N\to\infty} \frac{L(F_q)}{E[e^{-\Sigma F_q}]} = e^{\sigma(1-\sqrt{\rho})F_q+\sigma^2(1-\rho)/2}$$

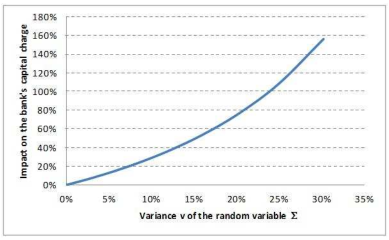


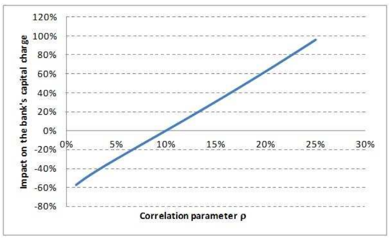
Cell risk dispersion

- **Analytical model with individual cell risk dispersion** $L_i = e^{-\sigma_i(\sqrt{\rho}F+\sqrt{1-\rho}\varepsilon_i)}$

- **Closed-form solution for the bank's loss when the number of cells goes to infinity**

$$L(F) = \frac{1}{\sqrt{1-(1-\rho)v}} e^{-\sigma\sqrt{\rho}F+\sigma^2(1-\rho)/2+\frac{\rho}{2}\frac{\left((1-\rho)\sigma-\sqrt{\rho}F\right)^2}{1-(1-\rho)v}}$$



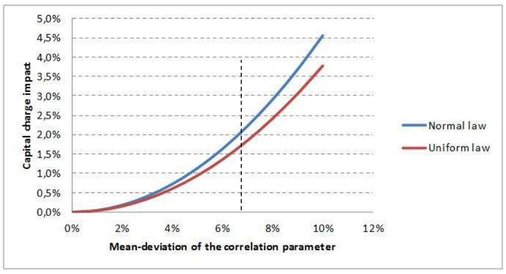


Correlation dispersion is not critical

- Analytical model with correlation dispersion $L(F) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} L_i = E\left[e^{-\sigma BF + \sigma^2(1-B^2)/2} \middle| F\right] = \int_{-\infty}^{\infty} dx\, f(x) e^{-x\sigma F + (1-x^2)\sigma^2/2}$

- As the correlation parameters are linked to the beta, their variances are linked as well

$$\rho_{ij} = \beta_i \cdot \beta_j \quad \Longrightarrow \quad w = \sqrt{\beta^4 + var(\rho_{ij})} - \beta^2$$



NEW RESULTS ON THE CLASSIFICATION PROBLEM

Classification invariance (1/2)

ASSUMPTIONS

- Homogeneous risk portfolio
- The shapes of the distributions don't change with the number N of cells
- The parameters scale with the number of cells
- The number of cells goes to infinity
- Cells risks are independant to each other

LOGNORMAL CASE

Bank's loss $L_N = \sum_{i=1}^{N} e^{\mu_N + \sigma_N X_i}$

Casymptotic classification invariance

$$\lim_{N\to\infty} E[L_N] = \lim_{N\to\infty} e^{\mu_N + \sigma_N^2/2} = a$$
$$\lim_{N\to\infty} var[L_N] = \lim_{N\to\infty} e^{2\mu_N + \sigma_N^2}\left(e^{\sigma_N^2} - 1\right) = b$$

Scaling of the parameters $\mu_N \sim -\frac{3}{2}\ln N$ and $\sigma_N \sim \sqrt{\ln N}$

Lindeberg's criterion $\lim_{N\to\infty} \frac{1}{s_N^2} \sum_{k=1}^{N} \int_{|Y_k - E(Y_k)| > \varepsilon s_N} [Y_k - E(Y_k)]^2 d\mathbb{P} = 0$

Domain of attraction of the normal distribution $\sigma_N < \sqrt{\frac{1}{2}\ln N}$

Classification invariance (2/2)

- **Domain of attraction of the bank's operational loss in the general case: Ben Arous, Bogachev, Molchanov Theorem**
- **There is a competition between the attraction of the normal distribution fixed point for the sum of i.i.d random variables and the divergence of the volatility parameter** σ_N
  - If the divergence is slow: domain of attraction of the normal distribution (Lindeberg's condition satisfied)
  - If the divergence is fast: domain of attraction of the fully asymetric Levy distribution.
- **Surprising results**
  - Fait tail (power law) distributions emerge from the classification invariance requirement
  - Distributions with finite variance are not in the domain of attraction of the normal distribution
  - Negative diversification occurs, even for uncorrelated cell risks
  - For correlated cells risks, classification invariance generates decorrelaltion among cells. The correlation parameter scales as:

$$\rho_N \sim K / \ln N$$

Conclusions

- **Average cell risk, cell risk dispersion and average correlations are critical parameters**
- **Regarding correlations**
  - they are very noisy
  - they seem low
  - Correlation dispersion is not a critical parameter
- **Diversification / negative diversification effects are not driven by correlations but by the shape of cell risk distributions**
  - Power laws and fat tails appear naturally when we require the classification invariance
  - Negative diversification may appear for large numbers of cells in the model
- **Analytical models have some vertues**
  - Avoid the black box feeling of the full statistical / Monte-Carlo approach
  - They embed very few specifications and lead to general results

- **The portfolio approach for operational risk is still unexplored, and we need to rethink the current approach to take into account of the scarcity of data**

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