Credit Value at Risk (CVaR)

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Banks take inevitably a lot of illiquid assets in their balance sheet. Therefore, they cannot easily manage or hedge their loan portfolios and have to hold most of the assets to maturity. The consequence is that banks expect and anticipate some losses. The survival of the bank is contingent to having losses lower than the revenues and the provisions. However, in the unfavorable cases, the portfolio will suffer larger losses than expected, and the risk of loss on the risky assets of the portfolio is used to measure the bank's cost of doing its business.

The sources of unanticipated losses are twofold. Firstly, even if the probability of default of an obligor is non zero, obligor defaults are generally not anticipated and may cause large losses on a loan portfolio when they occur. The second source of unanticipated risk are unexpected credit migrations, that make future defaults of the obligors more probable. Because of unanticipated sources of risk, the final loss of the portfolio is going to fluctuate around its expected value. There exist many risk measures are we distinguish between two regimes of risk :

- Unexpected losses are not anticipated, though predictable, but the bank is able to absorb them in the normal course of doing business.
- Extreme losses are highly rare, though not improbable, and the bank must be able to survive and remain solvent in those extreme and stressed conditions.

The sum of the expected loss and the extreme losses is the amount that the bank has to set aside in order to maintain its solvency, should extreme losses occur. We see that the quantification of extreme losses is useful for measuring the capital reserve which acts as a protection against insolvency. This capital reserve is known as economic capital, and we refer to the article "Risk Adjusted Return On Risk Adjusted Capital" in the encyclopedia.

The methodology adopted for measuring credit risk over the bank's portfolio is called Credit Value at Risk (CvaR). It is rather simple : the relationship between a loss level and its probability of occurrence is called the loss probability distribution. The goal is to estimate the loss level that is going to occur in a small fraction of the cases. If the losses are larger than this threshold, the bank defaults. The cornerstone of this methodology is the knowledge of the probability distribution of the bank's portfolio. This means that the probability that the bank's portfolio suffers losses larger than the sum of expected and unexpected losses is equal to the confidence level, let's say 99.9%. This is precisely the definition of Credit Value at Risk (CvaR).

$$P\left[L \leq CVaR_{99.9\%}\right] = 99.9\% \tag{1}$$

The 99.9% threshold is interpreted as the survival probability of the bank upon the time horizon considered, meaning that its default probability is 0.1%. This threshold depends of course on the rating aimed by the bank. The following table gives an indication of the confidence level required for a given rating target with the assumption of 2 years and a half portfolio maturity :

Rating target	Confidence level
AAA	99,97%
AA+	99,95%
AA	99,93%
AA-	99,90%
A+	99,87%
A	99,83%
A-	99,73%
BBB+	99,48%
BBB	99,16%
BBB-	98,25%
BB+	96,60%
BB	94,10%
BB-	91,33%

There are essentially two ways to compute CvaR. Contrary to the market VaR, historic VaR obtained by applying to the portfolio a set of scenarios that have occurred in the past is impossible to compute in the case of CvaR. Indeed, it is worthless to consider a historical scenario of default for the present portfolio since the defaulted obligors are no longer in the loan portfolio ! The two methods to estimate CvaR are parametric VaR and Monte Carlo simulations.

The most popular formula of parametric CVaR is to assume that economic capital is a multiple of the standard deviation of the credit losses. The parametric CvaR depends on the standard deviation of the loss on each line of the portfolio and on the correlations between each line. We call σ_i the unitary standard deviation of line *i* and ρ_{ij} the loss correlation between line *i* and line *j*. We get :

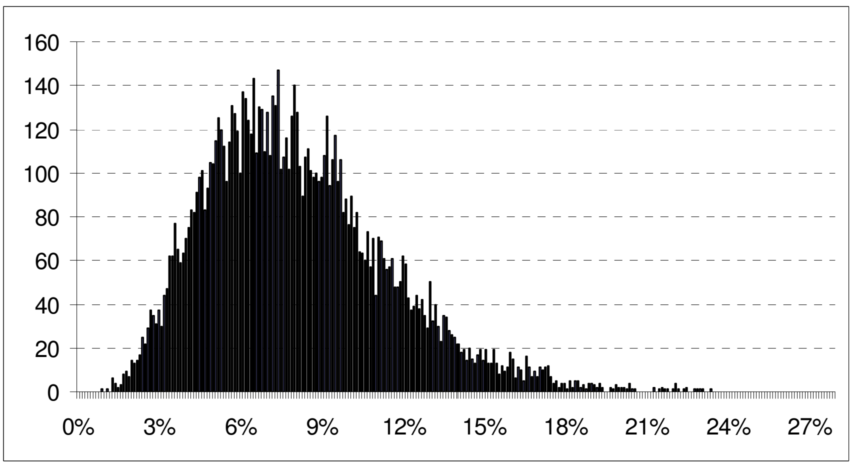
$$EC = k\sqrt{\sum_i \sigma_i^2 + 2\sum_{i<j} \rho_{ij}\sigma_i\sigma_j} \qquad (2)$$

The coefficient k is called the capital multiplier and is calibrated in order to be consistent with the 99.9% threshold. An estimate of the loss volatilities leads to the CvaR straightforwardly. This assumption is exact in the case of normal distributions, but remains incorrect for skewed distributions. However the great advantage of this method applied to credit risk is to get a parametric formula of the CvaR. The parametric formulation leads also straightforwardly to estimate of the marginal contribution and incremental capital of any line of the portfolio. On their side, the loss volatilities can easily be estimated analytically ; as an example, we can modelise the default of an obligor with a binomial law with default probability *p*. If the loss given default is 100%, the expected loss is going to be equal to *p*, and the loss volatility is $\sqrt{p(1-p)}$.

The second method consists in generating the loss distribution by direct Monte-Carlo simulations and to find the loss level corresponding to the confidence level 99.9%. The Monte-Carlo simulation is generated in several steps.

- Estimate the defaults and losses over each obligor of the portfolio. It consists in assigning a rating and a loss given default to any obligor.
- Estimate the dependence between obligors. In practice, either we determine pairwise asset correlations or obligors are assigned an industry sector and a country, and we determine industry and country correlations.
- Generate the correlated defaults and loss given defaults. Take into account guarantees, collaterals…
- Compute the losses at the transaction level and add them at the portfolio level.
- By repeating the above steps a large number of times, compute the loss distribution and the relevant indicators.

We are not going to give some details about the Monte-Carlo simulations. We refer for that to the article "Monte-Carlo simulations" in this encyclopedia. We have run a 3 years BB homogeneous portfolio simulation 10 000 times the loss distribution is shown in figure 1.



We find a Monte-Carlo CVaR equal to 22.1%. The parametric CVaR is equal to 17.8%. We see that Monte-Carlo simulations, though very time consuming generally lead to significantly different (and reliable !) results compared to the parametric method. This would be emphasized for marginal contributions.

VaR in general, and CvaR in particular, have been widely adopted as a risk measure or capital requirement, but is has been questioned for several years now. Indeed, VaR suffers some particular deficiencies. For instance, merging two portfolio can lead in some circumstances to a VaR larger than the sum of the VaR of initial portfolios in spite of an expected diversification. Many authors rather recommend to use more appropriate risk measures, and in particular the conditional excess $E\left[L \middle| L > VaR_{99.9\%}\right]$ has been shown to satisfy all the risk measure requirements

Further reading

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