COST OF ADDING A GUARANTEE TO A FUND

INTRODUCTION

We consider a fund (risky asset) having a lognormal price dynamics with drift μ and volatility σ. The expected return on the fund already integrates running fees f. We aim at selling a new fund invested into this risky asset, but with a guarantee at maturity *T*. This guarantee may be a capital guarantee or a performance guarantee.

The new fund is invested into the risky asset and a put on the risky asset. The case of a performance guarantee is more complex to study, because the higher the guaranteed performance, the lower the performance of the new fund. We show in this case that the price of the put of guarantee is the solution of a self-consistent equation.

SELF-CONSISTANCE EQUATION

We assume that the new fund benefits from a guaranteed performance of Eonia-x at maturity *T*. Let $R_0 = 100$ the initial wealth that the client invests in the new fund. We call P the price of the put of guarantee as a percentage of R_0, and m is the upfront margin the bank retains for selling the put option. At inception ($t = 0$), the client invests $R_0(1 - P - m)$ in the risky asset and $P.R_0$ for the put option. The put strike is equal to:

$$Strike = R_0\, e^{\int_0^T (r_s - x)\, ds}$$

We call S_t the price of the risky asset at time t and Q the risk-neutral probability; The put price is the solution of the following self-consistent equation:

$$P.R_0 = E^Q\left[e^{-\int_0^T r_s\, ds} \left(R_0\, e^{\int_0^T (r_s - x)\, ds} - R_0 (1 - P - m) \frac{S_T}{S_0} \right)^+ \right]$$

We define the forward price of the risky asset F_t equal to:

$$F_t = \frac{S_t}{S_0} e^{-\int_0^t r_s ds}$$

Under the forward-neutral probability, the forward price dynamics is:

$$\frac{dF_t}{F_t} = \sigma\, dW_t \qquad (1)$$

The put price is then the solution of the self-consistent equation:

$$P = E^{Q}\left[\left(e^{-xT} - (1-P-m)F_T\right)^{+}\right] \tag{2}$$

We can rewrite this equation by using the risky asset dynamics (1):

$$\begin{aligned} P &= e^{-xT}\ N(-d_2) - (1-P-m)\, N(-d_1) \\ d_{1,2} &= \frac{\ln(1-P-m) + \left(x \pm \sigma^2/2\right)T}{\sigma\sqrt{T}} \end{aligned} \tag{3}$$

Equation (3) is a fixed-point equation of the form $P = f(P)$. The function $f(P) - P$ is a positive decreasing function the interval $[0;+\infty[$ and crosses the horizontal axis under some conditions. This fixed-point equation then has only one solution that is obtained by iterating the sequence $u_{n+1} = f(u_n)$ and $u_0 = 0$.

With the following parameters:

- T = 1 year
- σ = 30 bppa
- m = 0
- x = 10 bp

The price of the put is: XXX bp

TARGET RETURN OF THE NEW FUND

The base scenario is that the risky asset performs correctly and the put remains out of the money. In this scenario, the yearly return of the new fund is:

$$\frac{1}{T}\ln\left(\frac{R_0\,(1-P-m)\,e^{\mu T}}{R_0}\right) \approx \mu - (P+m)/T \tag{4}$$

Equation (4) illustrates perfectly the impact of the put price on the fund performance.